Quantitative Aptitude Cheat Sheet

Formulae and Fundas

Ramandeep Singh

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Number system

Natural Numbers: 1, 2, 3, 4…..
Whole Numbers: 0, 1, 2, 3, 4…..
Integers: ….-2, -1, 0, 1, 2 …..

Rational Numbers: Any number which can be expressed as a ratio of two integers for example a p/q format where ‘p’ and ‘q’ are integers. Proper fraction will have (p<q) and improper fraction will have (p>q)

Factors: A positive integer ‘f’ is said to be a factor of a given positive integer 'n' if f divides n without leaving a remainder. e.g. 1, 2, 3, 4, 6 and 12 are the factors of 12.

Prime Numbers: A prime number is a positive number which has no factors besides itself and unity.

Composite Numbers: A composite number is a number which has other factors besides itself and unity.

Factorial: For a natural number 'n', its factorial is defined as: n! = 1 x 2 x 3 x 4 x .... x n (Note: 0! = 1)

Absolute value: Absolute value of x (written as |x|) is the distance of 'x' from 0 on the number line. |x| is always positive. |x| = x for x > 0 OR -x for x < 0

Laws of Indices

$a^m \times a^n = a^m + n$

$a^m \div a^n = a^m \;\; n$

$a^{(m)n} = a^{mn}$

$a^{1/m} = \sqrt[m]{a}$

$a \;\; ^m = \frac{1}{a^m}$

$a^{m/n} = \sqrt[n]{a^m}$

$a^0 = 1$

HCF and LCM

For two numbers, HCF x LCM = product of the two.

HCF of Fractions = $\frac{HCF\ of\ Numerator}{LCM\ of\ Denominator}$

LCM of Fractions = $\frac{LCM\ of\ Numerator}{HCF\ of\ Denominator}$

Relatively Prime or Co-Prime Numbers: Two positive integers are said to be relatively prime to each other if their highest common factor is 1.

Divisibility Rules

A number is divisible by: 2, 4 & 8 when the number formed by the last, last two,last three digits are divisible by 2,4 & 8 respectively. 3 & 9 when the sum of the digits of the number is divisible by 3 & 9 respectively.

11 when the difference between the sum of the digits in the odd places and of those in even places is 0 or a multiple of 11.

6, 12 & 15 when it is divisible by 2 and 3, 3 and 4 & 3 and 5 respectively.

7, if the number of tens added to five times the number of units is divisible by 7.

13, if the number of tens added to four times the number of units is divisible by 13.

19, if the number of tens added to twice the number of units is divisible by 19.

Algebraic Formulae

$a^3 \pm b^3$ = (a ± b) ($a^2 \mp$ ab + b^2). Hence, $a^3 \pm b^3$ is divisible by ((a ± b) and ($a^2 \pm$ ab + b^2).

$a^n \;\; b^n$ = (a b) ($a^{n \;\; 1}$ + $a^{n \;\; 2}$b + $a^{n \;\; 3}b^2$ + … + $b^{n \;\; 1}$) (for all n). Hence, $a^n \;\; b^n$ is divisible by a b for all n.

$a^n \;\; b^n$ = (a + b) ($a^{n \;\; 1}$ + $a^{n \;\; 2}$b + $a^{n \;\; 3}b^2$ + … $b^{n \;\; 1}$) (n even). Hence, $a^n \;\; b^n$ is divisible by a + b for even n.

$a^n + b^n$ = (a + b) (a^{n-1} $a^{n-2}b + a^{n-3}b^2$ + … + b^{n-1}) (n odd). Hence, $a^n + b^n$ is divisible by a + b for odd n.

$a^3 + b^3 + c^3$ 3abc = (a + b + c) $(a^2 + b^2 + c^2$ ab ac bc) Hence, $a^3 + b^3 + c^3$ = 3abc if a + b +c = 0

Average

Simple Average $= \frac{Sum\,of\,elements}{Number\,of\,elements}$

Weighted Average = $\frac{W1 \times 1 + W2x2 + \cdots + Wn \times n}{W1 + W2 + \cdots + Wn \times n}$

Arithmetic Mean = (a1 + a2 + a3 …an)/n

Geometric Mean = $\sqrt[n]{a1\ a2 \ldots an}$

Harmonic Mean = $\frac{n}{\frac{1}{x1}+\frac{1}{x2}+\cdots+\frac{1}{xn}}$

Percentage

Fractions and their percentage equivalents:

Fraction	Percentage	Fraction	Percentage
1/2	50%	1/9	11.11%
1/3	33.33%	1/10	10%
1/4	25%	1/11	9.09%
1/5	20%	1/12	8.33%
1/6	16.66%	1/13	7.69%
1/7	14.28%	1/14	7.14%
1/8	12.5%	1/15	6.66%

Interest

Amount = Principal + Interest
Simple Interest = PNR/100

Compound Interest = P $\left(1 + \frac{r}{100}\right)^n - P$

Population formula P' = P $\left(1 \pm \frac{r}{100}\right)^n$

Depreciation formula = initial value × $\left(1 - \frac{r}{100}\right)^n$

Growth and Growth Rates

Absolute Growth = Final Value – Initial Value
Growth rate for one year period = $\frac{Final\,value - Initial\,value}{Initial\,value}$ × 100
SAGR or AAGR = $\frac{Final\,value - Initial\,value}{No.of\,years}$ × 100

CAGR = $\left(\frac{Final\,value - Initial\,value}{Initial\,value}\right)^{\frac{1}{No.of\,years}}$ 1

Profit and Loss

%Profit/Loss = $\frac{Selling\,Price - Cost\,Price}{Initial\,value}$ x 100

In case false weights are used while selling,

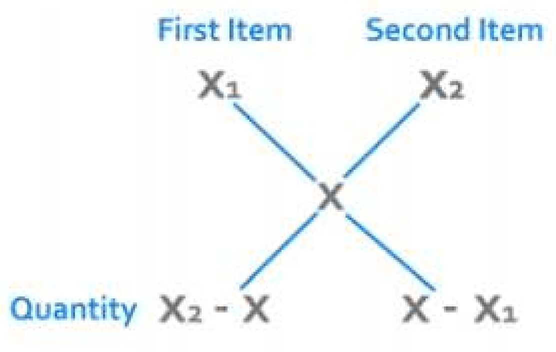
% Profit = $\left(\frac{Claimed\,Weight - Actual\,Weight}{Actual\,Weight}\right.$ 1) x 100

Discount % = $\frac{Marked\,Price - Selling\,Price}{Marked\,Price}$ × 100

Mixtures and Alligations

Alligation – The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture

$\frac{Quantity\,of\,first\,item}{Quantity\,of\,second\,item} = \frac{x_2 - x}{x - x_1}$



Ratio and Proportion

Compounded Ratio of two ratios a/b and c/d is ac/bd,
Duplicate ratio of a : b is $a^2 : b^2$
Triplicate ratio of a : b is $a^3 : b^3$
Sub-duplicate ratio of a : b is $\sqrt{a}$: $\sqrt{b}$
Sub-triplicate ratio of a : b is $\sqrt[3]{a} : \sqrt[3]{b}$
Reciprocal ratio of a : b is b : a

Componendo and Dividendo

If $\frac{a}{b} = \frac{c}{d}$ & a ≠ b then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

Four (non-zero) quantities of the same kind a, b, c, d are said t o be in proportion if a/b = c/d.
The non-zero quantities of the same kind a, b, c, d are said be in continued proportion if a/b = b/c = c/d.

Proportion

a, b, c , d are said to be in proportion if $\frac{a}{b} = \frac{c}{d}$

a, b , c, d are said to be in continued proportion if $\frac{a}{b} = \frac{b}{c} = \frac{c}{d}$

Time Speed and Distance

Speed = Distance / Time

1 kmph = 5/18 m/sec; 1 m/sec = 18/5 kmph

$$Speed_{Avg} = \frac{Total\ distance\ covered}{Total\ time\ taken} = \frac{d_1 + d_2 + d_3 \ldots d_n}{t_1 + d_1}$$

If the distance covered is constant then the average speed is Harmonic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

$$Speed_{Avg} = \frac{n}{\frac{1}{s_1}+\frac{1}{s_2}+\frac{1}{s_3}\ldots\frac{1}{s_n}}$$

$Speed_{Avg} = \frac{2s_1s_2}{s_1+s_2}$ (for two speeds)

If the time taken is constant then the average speed is Arithmetic Mean of the values ($s_1, s_2, s_3 \ldots s_n$)

$$Speed_{Avg} = \frac{s_1, s_2, s_3 \ldots s_n}{n}$$

$Speed_{Avg} = \frac{s_1+s_2}{2}$ (for two speeds)

Races & Clocks

Linear Races

Winner's distance = Length of race

Loser's distance = Winner's distance – (beat distance + start distance)

Winner's time = Loser's time – (beat time + start time)

Deadlock / dead heat occurs when beat time = 0 or beat distance = 0

Circular Races

Two people are running on a circular track of length L with speeds a and b in the same direction

Time for 1st meeting = $\frac{L}{a-b}$

Time for 1st meeting at the starting point = LCM ($\frac{L}{a}$, $\frac{L}{b}$)

Clocks: To solve questions on clocks, consider a circular track of length 360°. The minute hand moves at a speed of 6° per min and the hour hand moves at a speed of ½° per minute.

Time and Work

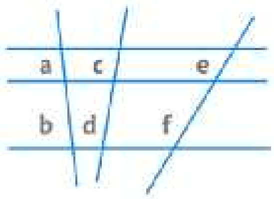
If a person can do a certain task in t hours, then in 1 hour he would do 1/t portion of the task.

A does a particular job in 'a' hours and B does the same job in 'b' hours, together they will take $\frac{ab}{a+b}$ hours

A does a particular job in 'a' hours more than A and B combined whereas B does the same job in 'b' hours more than A and B combined, then together they will take $\sqrt{ab}$ hours to finish the job.

Geometry

Lines and Angles



- Sum of the angles in a straight line is 180°
- Vertically opposite angles are always equal.
- If any point is equidistant from the endpoints of a segment, then it must lie on the perpendicular bisector.

- When two parallel lines are intersected by a transversal, corresponding angles are equal, alternate angles are equal and co-interior angles are supplementary. (All acute angles formed are equal to each other and all obtuse angles are equal to each other)

Fact

- The ratio of intercepts formed by a transversal intersecting three parallel lines is equal to the ratio of corresponding intercepts formed by any other transversal.

$$\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$$

Triangles

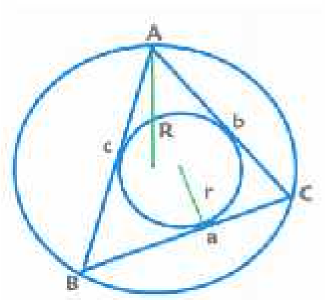
- Sum of interior angles of a triangle is 180° and sum
  of exterior angles is 360°.

- Exterior Angle = Sum of remote interior angles.

- Sum of two sides is always greater than the third side and the difference of two sides is always lesser than the third side.

- Side opposite to the biggest angle is longest and the side opposite to the smallest angle is the shortest.

Area of a triangle:



= ½ x Base x Height
= ½ x Product of sides x Sine of included angle
= $\sqrt{s(s-a)(s-b)(s-c)}$ here s is the semi perimeter
[s = (a+b+c)/2]
= r x s [r is radius of incircle]
= $\frac{abc}{4R}$ [R is radius of circumcircle]

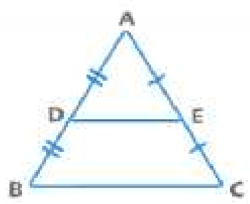
Median
A Median of a triangle is a line segment joining a vertex to the midpoint of the opposing side. The three medians intersect in a single point, called the Centroid of the triangle. Centroid divides the median in the ratio of 2:1

Altitude
An Altitude of a triangle is a straight line through a vertex and perpendicular to the opposite side or an extension of the opposite side. The three altitudes intersect in a single point, alled the Orthocenter of the triangle.

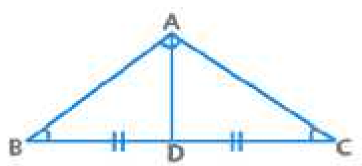
Perpendicular Bisector
A Perpendicular Bisector is a line that forms a right angle with one of the triangle's sides and intersects that side at its midpoint. The three perpendicular bisectors intersect in a single point, called the Circumcenter of the triangle. It is the center of the circumcircle which passes through all the vertices of the triangle.

Angle Bisector
An Angle Bisector is a line that divides the angle at one of the vertices in two equal parts. The three angle bisectors intersect in a single point, called the Incenter of the triangle. It is the center of the incircle which touches all sides of a triangle.

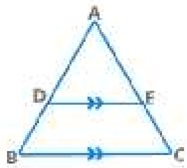
Theorems
Mid Point Theorem: The line joining the midpoint of any two sides is parallel to the third side and is half the length of the third side.



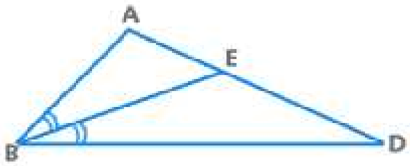
Apollonius' Theorem: AB^2 + AC^2 = 2(AD^2 + BD^2)



Basic Proportionality Theorem: If DE || BC, then AD/DB = AE/EC

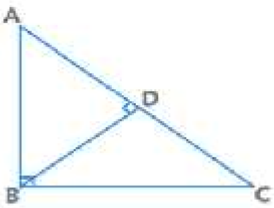


Interior Angle Bisector Theorem: AE/ED = BA/BD

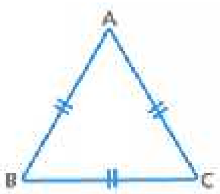


Special Triangles
Right Angled Triangle:

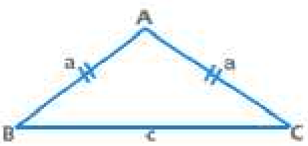
ABC ≈ ADB ≈ BDC
BD^2 = AD x DC and AB x BC = BD X DC



Equilateral Triangle:
All angles are equal to 60°. All sides are equal also.

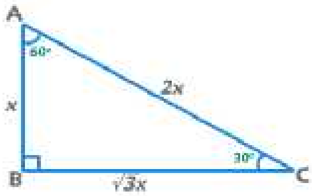


Isosceles Triangle:
Angles equal to opposite sides are equal.
Area = $\frac{c}{4}\sqrt{4a^2 - c^2}$



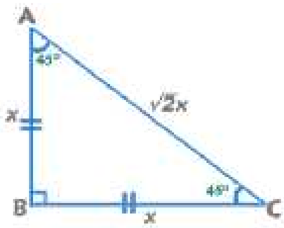
30°-60°-90° Triangle

Area = $\frac{\sqrt{3}}{2} \times x^2$

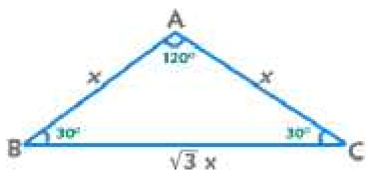


45°-45°-90° Triangle

Area = $\frac{x^2}{2}$



30°-30-120° Triangle
Area = $\frac{\sqrt{3}}{4} \times x^2$



Similarity of Triangles

Two triangles are similar if their corresponding angles are congruent and corresponding sides are in proportion.

Tests of similarity: (AA / SSS / SAS)

- For similar triangles, if the sides are in the ratio of a:b
- Corresponding heights are in the ratio of a:b
- Corresponding medians are in the ratio of a:b
- Circumradii are in the ratio of a:b
- Inradii are in the ratio of a:b
- Perimeters are in the ratio of a:b
- Areas are in the ratio a2 : b2

Congruency of Triangles
Two triangles are congruent if their corresponding sides and angles are congruent.

Tests of congruence: (SSS / SAS / AAS / ASA)
All ratios mentioned in similar triangle are now 1:1

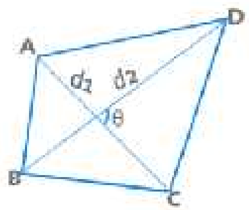
Polygons

- Sum of interior angles = (n - 2) x 180° = (2n - 4) x 90°
- Sum of exterior angles = 360°
- Number of diagonals = $^nC_2 - n = \frac{n(n-3)}{2}$
- Number of triangles which can be formed by the vertices = $^nC_3$

Regular Polygon :

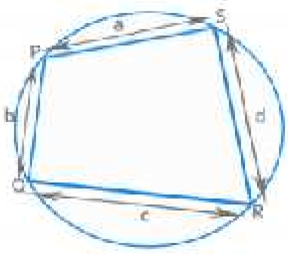
- If all sides and all angles are equal, it is a regular polygon.
- All regular polygons can be inscribed in or circumscribed about a circle.
- Area = ½ × Perimeter × Inradius {Inradius is the perpendicular from centre to any side}
- Each Interior Angle = $\frac{(n-2)180°}{n}$; Exterior = $\frac{360°}{n}$

Quadrilaterals :



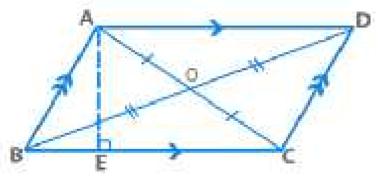
- Sum of the interior angles = Sum of the exterior angles = 360°
- Area for a quadrilateral is given by $\frac{1}{2}$ $^d1$ $^d2$ Sin θ

Cyclic Quadrilateral



- If all vertices of a quadrilateral lie on the circumference of a circle, it is known as a cyclic quadrilateral.
- Opposite angles are supplementary
- Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semi perimeter s = $\frac{a+b+c+d}{2}$

Parallelogram



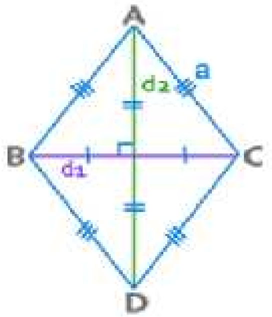
- Opposite sides are parallel and congruent.
- Opposite angles are congruent and consecutive angles are supplementary.
- Diagonals of a parallelogram bisect each other.
- Perimeter = 2(Sum of adjacent sides);
- Area = Base x Height = AD x BE

Facts

- Each diagonal divides a parallelogram in two triangles of equal area.
- Sum of squares of diagonals = Sum of squares of four sides
    - $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$

- A Rectangle is formed by intersection of the four angle bisectors of a parallelogram.

Rhombus



- A parallelogram with all sides equal is a Rhombus. Its diagonals bisect at 90°.
- Perimeter = 4a; Area = $\frac{1}{2}$ $d_1 d_2$
- Area = d x $\sqrt{a^2 - (\frac{d}{2})^2}$

Rectangle

A parallelogram with all angles equal (90°) is a Rectangle. Its diagonals are congruent.

- Perimeter = 2(l+b)
- Area = lb

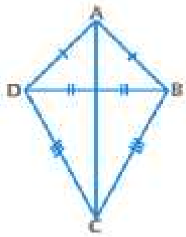
Square

A parallelogram with sides equal and all angles equal is a square. Its diagonals are congruent and bisect at 90°.

- Perimeter = 4a
- Area = a^2
- Diagonals = a $\sqrt{2}$

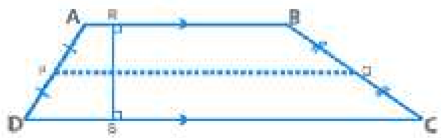
Fact: From all quadrilaterals with a given area, the square has the least perimeter. For all quadrilaterals with a given perimeter, the square has the greatest area.

Kite



- Two pairs of adjacent sides are congruent.
- The longer diagonal bisects the shorter diagonal at 90°.
- Area = $\frac{product\ of\ diagonals}{2}$

Trapezium / Trapezoid



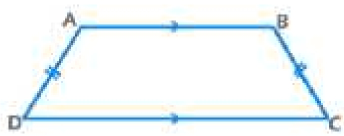
- A quadrilateral with exactly one pair of sides parallel is known as a Trapezoid. The parallel sides are known as bases and the non-parallel sides are known as lateral sides.
- Area = $\frac{1}{2}$ × (Sum of parallel sides) × Height
- Median, the line joining the midpoints of lateral sides, is half the sum of parallel sides.

Fact

- Sum of the squares of the length of the diagonals = Sum of squares of lateral sides + 2 Product of bases.

$AC^2 + BD^2 = AD^2 + BC^2$ + 2 × AB × CD

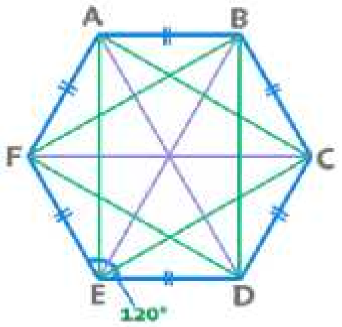
Isosceles Trapezium



The non-parallel sides (lateral sides) are equal in length. Angles made by each parallel side with the lateral sides are equal.

Facts: If a trapezium is inscribed in a circle, it has to be an isosceles trapezium. If a circle can be inscribed in a trapezium, Sum of parallel sides = Sum of lateral sides.

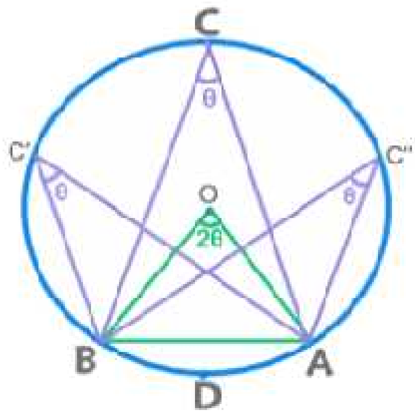
Hexagon (Regular)



- Perimeter = 6a; Area = $\frac{3\sqrt{3}}{2} \times a^2$
- Sum of Interior angles = 720°.
- Each Interior Angle = 120°. Exterior = 60°
- Number of diagonals = 9 {3 big and 6 small}
- Length of big diagonals (3) = 2a
- Length of small diagonals (6) = $\sqrt{3}$ a
- Area of a Pentagon = 1.72 a^2
- Area of an Octagon = 2($\sqrt{2}$ + 1) a^2

Facts: A regular hexagon can be considered as a combination of six equilateral triangles. All regular polygons can be considered as a combination of 'n' isosceles triangles.

Circles

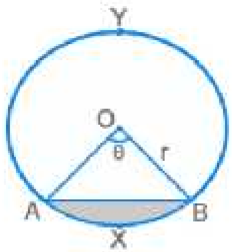


Diameter = 2r; Circumference = 2πr; Area = πr^2

Chords equidistant from the centre of a circle are equal. A line from the centre, perpendicular to a chord, bisects the chord. Equal chords subtend equal angles at the centre.

The diameter is the longest chord of a circle. A chord /arc subtends equal angle at any point on the circumference and double of that at the centre.

Chords / Arcs of equal lengths subtend equal angles.

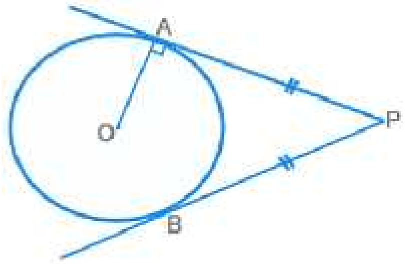


Chord AB divides the circle into two parts: Minor Arc AXB and Major Arc AYB

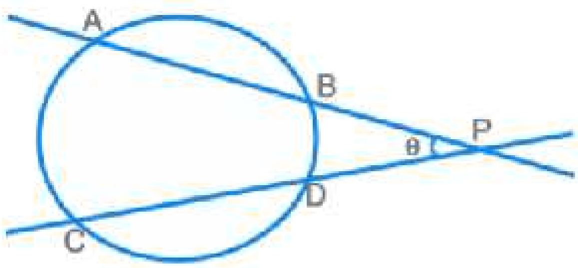
- Measure of arc AXB = ∠AOB = θ
- Length (arc AXB) = $\frac{\theta}{360°} \times 2\pi r$
- Area (sector OAXB) = $\frac{\theta}{360°} \times \pi r^2$
- Area of Minor Segment = Shaded Area in above figure

- Area of Sector OAXB - Area of ΔOAB
- $r^2 [\frac{\pi\theta}{360°} \quad \frac{sin\theta}{2}]$

Properties of Tangents, Secants and Chords

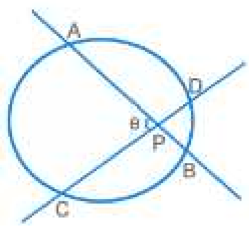


The radius and tangent are perpendicular to each other. There can only be two tangents from an external point, which are equal in length PA = PB



PA × PB = PC × PD

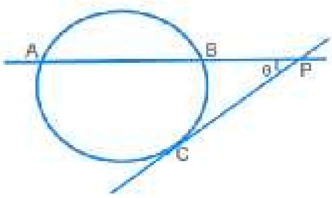
= $\frac{1}{2}$ [m(Arc AC) – m(Arc BD)]



PA × PB = PC × PD

= $\frac{1}{2}$ [m(Arc AC) + m(Arc BD)]

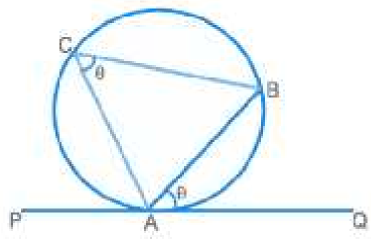
Properties



PA × PB = PC^2

= $\frac{1}{2}$ [m(Arc AC) - m(Arc BC)]

Alternate Segment Theorem

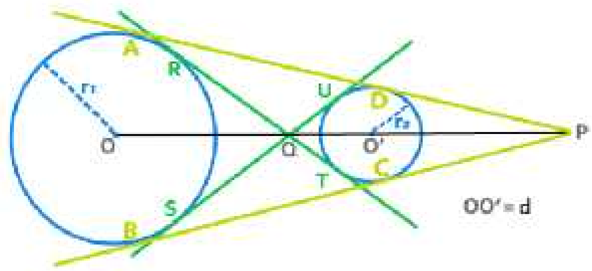


The angle made by the chord AB with the tangent at A (PQ) is equal to the angle that it subtends on the opposite side of the circumference.

∠BAQ = ∠ACB

Common Tangents

Two Circles	No. of Common Tangents	Distance Between Centers (d)
One is completely inside other	0	< r1 r2
Touch internally	1	= r1 r2
Intersect	2	r1 r2 < d < r1 + r2
Touch externally	3	= r1 + r2
One is completely outside other	4	> r1 + r2



- Length of the Direct Common Tangent (DCT) AD = BC = $\sqrt{d^2 - (r1 - r2)^2}$
- Length of the Transverse Common Tangent (TCT)

RT = SU = $\sqrt{d^2 - (r1 + r2)^2}$

Venn Diagram: A venn diagram is used to visually represent the relationship between various sets. What do each of the areas in the figure represent?

I – only A; II – A and B but not C; III – Only B; IV – A and C but not B; V – A and B and C; VI – B and C but not A; VII – Only C

n(A∪B∪C) = n(A) + n(B) + n(C) — n(A∩B) — n(A∩C) - n(B∩C) + n(A∩B∩C)

Permutation and Combination

When two tasks are performed in succession, i.e., they are connected by an 'AND', to find the total number of ways of performing the two tasks, you have to MULTIPLY the individual number of ways. When only one of the two tasks is performed, i.e. the tasks are connected by an 'OR', to find the total number of ways of performing the two tasks you have to ADD the individual number of ways.

Eg: In a shop there are‘d’ doors and ‘w’ windows.

Case1: If a thief wants to enter via a door or window, he can do it in – (d+w) ways.

Case2: If a thief enters via a door and leaves via a window, he can do it in – (d x w) ways.

Linear arrangement of ‘r’ out of ‘n’ distinct items (^np_r):
The first item in the line can be selected in 'n' ways AND the second in (n — 1) ways AND the third in (n — 2) ways AND so on. So, the total number of ways of arranging 'r' items out of 'n' is

(n)(n - 1)(n — 2)...(n - r + 1) = $\frac{n!}{(n-r)!}$

Circular arrangement of 'n' distinct items: Fix the first item and then arrange all the other items linearly with respect to the first item. This can be done in (n − 1)! ways.

Probability

P(A) = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ outcomes}$

For Complimentary Events: P(A) + P(A’) = 1
For Exhaustive Events: P(A) + P(B) +P(C)… = 1

Addition Rule:
P (A U B) = P(A) + P(B) – P(A∩B)

For Mutually Exclusive Events P(A∩ B) = 0
P (A U B) = P(A) + P(B)

Multiplication Rule: P(A∩B) = P(A) P(B/A) = P(B) P(A/B)

For Independent Events P(A/B) = P(B) and P(B/A) = P(B)
P(A∩B) = P(A).P(B)
P(A U B) = P(A) + P(B) – P(A).P(B)

Sequence, Series and Progression

Arithmetic Progression

$a_n = a_1 + (n - 1)d$

$S_n = \frac{n}{2}(a_1 + a_n) = \frac{n}{2}(2a_1 + (n - 1)d)$

Geometric Progression

$a_n = ar^{n-1}$

$S_n = \frac{a(1 - r^n)}{1 - r}$

Sum till infinite terms = $\frac{a}{1-r}$ (valid only when r < 1)