Problems on Trains with Solutions - PDF

18 km/hour = 5 metre/second, can be used to convert the speed given in km/hour to metre/second or vice - versa.

3 km/hour = (3 × 5 / 18) metre/second

and

3 metre/second = (3 × 18 / 5) km/hour

#2
If a train of length "L" at a speed "v "passes another object of length " *l*" at a speed x in time "t", then
t = time to cross the object = Total Distance / Relative Speed = L + *l* / v - x

- If object is stationary, then x = 0
- If object is opposite, then put x = -x
- and v - x making after is (v + x)

A train is said to have crossed an object (stationary or moving) only when the last coach (end) of the train crosses the said object completely. It implies that the total length of the train has crossed the total length of the object.

> *Hence, the distance covered by the train = length of train + length of object*

Basic Formula

1. Time = Distance / Speed
2. Time to cross an object moving in the direction of train =

Time to cross an object moving in the direction of train

$$= \frac{\textit{Length of train} + \textit{Length of Object}}{\textit{Speed of train} - \textit{Speed of object}}$$

Assume :
Length of Train = L_t
Length of Object = *L*
Speed of Train = V_t = time to cross
Speed of Object = V
Then basic formula can be represented as
$t = \dfrac{L_t + L}{V_t - V}$

Important Note

1. If the object is of negligible length, then put length of object, L = 0 (i.e. tree, man)
2. If the object is stationary, then put speed of object, V = 0
3. If the object is moving in opposite direction, then put (-) ve sign before V, so the denominator of the formula becomes

Different Types of Objects

$$V_t - (-V) = (V_t + V)$$

$$\text{Then } t = \frac{(L_t + L)}{(V_t + V)}$$

Case	Types of Objects	Time to Cross
1.	Object is stationary and is of negligible length e.g. train crosses lamp post, pole, standing man etc.	$t = \frac{\text{Length of Train}}{\text{Speed of Train}}$
2.	Object is stationary and is of some length, e.g. train crosses a bridge, a tunnel, platform, or another train at rest	$t = \frac{\text{Length of (Train+Object)}}{\text{Speed of Train}}$
3.	Object is moving and is of negligible length, e.g. train crosses a running man, a running car etc.	$t = \frac{\text{Length of Train}}{\text{Speed of (Train}-\text{Object)}}$
4.	Object is moving and has some length, e.g. train crosses another running train	$t = \frac{\text{Length of (Train+Object)}}{\text{Speed of (Train}-\text{Object)}}$

> *If the object is moving to opposite direction, then denominator becomes speed of (train + object)*

Two Trains Crossing Each Other in Both Directions

Two trains are crossing each other

Assume :
Length of One Train = L_1
Length of Second Train = L_2
They are crossing each other in opposite direction in t_1 seconds
They are crossing each other in same direction in t_2 seconds
Then
Speed of Faster Train = $\frac{L_1 + L_2}{2}\left[\frac{1}{t_1} + \frac{1}{t_2}\right]$
Speed of Slower Train = $\frac{L_1 + L_2}{2}\left[\frac{1}{t_1} - \frac{1}{t_2}\right]$

APPLICATIONS ON TRAINS.

1. How long will a train 130m long travelling at 40 km an hour take to pass a kilometer stone?

Solution: Time = $\frac{Total\ distance}{Speed} = \frac{0.130}{40}$ hr = $\frac{0.130 \times 60 \times 60}{40}$ = 11.7 sec.

2. How long will a train 60 m long travelling at 40 km an hour take to pass through a station whose platform is 90 m long?

Solution: Speed = 40 km/hr = 40 $\times \frac{5}{18}$ m/s

$\therefore$ Time = $\frac{(60+90)}{40 \times 5} \times 18 = \frac{150 \times 18}{40 \times 5}$ = 13.5 seconds.

Solution: 30km/hr = 30 $\times \frac{5}{18} = \frac{25}{3}$m/s

Length of train = 25 × 27 = × 225 = 112.5m

Length of train = $\frac{[illegible]}{3} \times \frac{[illegible]}{2} = \times \frac{[illegible]}{2} = 112.5m$

4. Find the length of a bridge which a train 130 m long, travelling at 45 km an hour, can cross in 30 secs.

Solution: 45 km/hr = $45 \times \frac{5}{18} = \frac{25}{2} = 12.5\frac{m}{s}$

Distance covered by the train in 30 seconds = $12.5 \times 30 = 375\,m$

∴ Length of bridge = 375 – 130 = 245 m.

5. The length of the train that takes 8 seconds to pass a pole when it turns at a speed of 36 km/hr is ____________________ metres.

Solution: 36 km/hr = $36 \times \frac{5}{18}$ = 10 m/s

Distance covered by train in 8 seconds= length of train = 8×10 = 80 m

6. A train 50 metres long passes a platform 100 metres long I 10 seconds. The speed of the train is _______________ km/hr.

Solution: Speed of train = $\frac{100+50}{10}$ = 15 m/s = $\frac{15 \times 18}{5}$ = 54 km/hr.

7. How many seconds will a train 60 m in length, travelling at the rate of 42 km an hour, take to pass another train 84 m long, proceeding in the same direction at the rate of 30 km an hour?

Solution: Relative Speed = 42 – 30 = 12 km/hr = $12 \times \frac{5}{18} = \frac{10}{3}$ m/s

Time = $\frac{Total\ length\ of\ both\ theh\ train}{Relative\ speed} = \frac{84+60}{\frac{10}{3}} = \frac{144 \times 3}{10}$ = 43. 2 seconds

8. A train 75 metres long overtook a person who was walking at the rate of 6 km an hour, and passed him in 7 $\frac{1}{2}$ seconds. Subsequently it overtook a second person, and passed him in 6 $\frac{3}{4}$ seconds. At what rate was the second travelling?

Solution: Relative Speed of train and the first person = $\frac{75}{\frac{15}{2}}$ = 10 m/s

= $10 \times \frac{18}{5}$ = 36 km/hr.

∴ Speed of train = 36 + 6 = 42 km /hr

Now, relative speed of train and 2nd person = $\frac{75}{27} \times 4$ m/s = $\frac{300}{27} \times \frac{18}{5}$ = 40 km/hr

∴ Speed of 2nd person = 42 - 40 = 2 km /hr.

Quicker Maths (Direct Formula): Speed of 2nd person = Relative Speed of train with respect of 1st person + Speed of first person – Relative speed of train with respect to 2nd person

$$= \left(\frac{75}{\frac{15}{2}} \times \frac{18}{5}\right) + 6 - \left(\frac{75}{27} \times 4 \times \frac{18}{5}\right) = 36 + 6 - 40 = 2 \text{ km/hr}$$

9. Two trains running at the rates 45 and 36 km an hour respectively, on parallel rails in opposite directions, are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rate as before, a person sitting in the faster train observes that he passes the other in 30 seconds. Find the lengths of the trains.

Solution: Relative Speed of two trains = 45 + 36 = 81 km/hr (When two train are moving in opposite directions)

$= 81 \times \frac{5}{18} = \frac{45}{2} = 22\frac{1}{2}$ m/s

∴ Length of both the trains = $\frac{45}{2} \times 8$ = 180 m.

Now, when two trains are moving in the same direction, the relative speed

$= 45 - 36 = 9$ km/hr $= \frac{9 \times 5}{18} = \frac{5}{2}$ m/s.

The man sitting in the faster train passes the length of the slower train in 30 seconds.

∴ Length of the slower train = $\frac{5}{2} \times 30 = 75\ m.$

∴ Length of the faster train = 180 – 75 = 105 m.

Quicker maths (Direct Formula):

Length of slower train = 30 $\times$ (Relative speed of two trains)= 30 (45 – 36) $\frac{5}{18}$ = 75 m

Length of faster train = Total length of both trains – length of slower train

= 8 (45 + 36) $\frac{5}{18}$ - 75

= 8 $\times$ $\frac{81 \times 5}{18}$ - 75 = 180 – 75 = 105 m.

10. Two trains measuring 100 m and 80 m respectively, run on parallel lines of rails. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the rates as before, the faster train passes the other in 18 seconds. Find the speed of the two trains in km per hour.

Solution: Quicker Maths (Direct Formula):

R_1 = Relative speed, when they are moving in the same direction

$= \frac{100+80}{18} = 10m/s$

R_2 = Relative Speed, when they are moving in opposite directions

$= \frac{100 + 80}{9} = 20$ m/s

Speed of faster train = $\frac{R_1 + R_2}{2} = \frac{10+20}{2}$ = 15 m/s

$= 15 \times \frac{18}{5} =$ 54 km/hr

Speed of slower train = $\frac{R_2 - R_1}{2} = \frac{20-10}{2} =$ 5 m/s

$= 5 \times \frac{18}{5} =$ 18 km/hr.