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Data Sufficiency Workbook

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a) Statement (i) ALONE is sufficient, but statement (ii) alone is not sufficient.
b) Statement (ii) ALONE is sufficient, but statement (i) alone is not sufficient.
c) BOTH statements TOGETHER are sufficient, but NEITHER statement ALONE is sufficient.
d) EACH statement ALONE is sufficient.
e) Statement (i) and (ii) TOGETHER are NOT sufficient to answer the question asked, and additional data are needed.

(1) If x and y are positive integers, is the following cube root an integer?

$$\sqrt[3]{x + y^2}$$

(i) $x = y^2(y-1)$
(ii) x = 2

(2) If w, x, y, and z are the digits of the four-digit number *N*, a positive integer, what is the remainder when *N* is divided by 9?

(i) w + x + y + z = 13
(ii) N + 5 is divisible by 9

(3) If x and y are distinct positive integers, what is the value of $x^4 - y^4$?

(i) $(y^2 + x^2)(y + x)(x - y) = 240$
(ii) $x^y = y^x$ and x > y

(4) If $z = x^n - 19$, is z divisible by 9?

(i) x = 10; n is a positive integer
(ii) z + 981 is a multiple of 9

(5) x is a positive integer; what is the value of x?

(i) The sum of any two positive factors of x is even
(ii) x is a prime number and x < 4

(6) x is an integer and x raised to any odd integer is greater than zero; is w - z greater than 5 times the quantity $7^{x-1} - 5^x$?

(i) z < 25 and $w = 7^x$
(ii) x = 4

(7) x is a positive integer greater than two; is $(x^3 + 19837)(x^2 + 5)(x – 3)$ an odd number?

(i) the sum of any prime factor of x and x is even
(ii) 3x is an even number

(8) If N, C, and D are positive integers, what is the remainder when D is divided by C?

(i) If D+1 is divided by C+1, the remainder is 5.
(ii) If ND+NC is divided by CN, the remainder is 5.

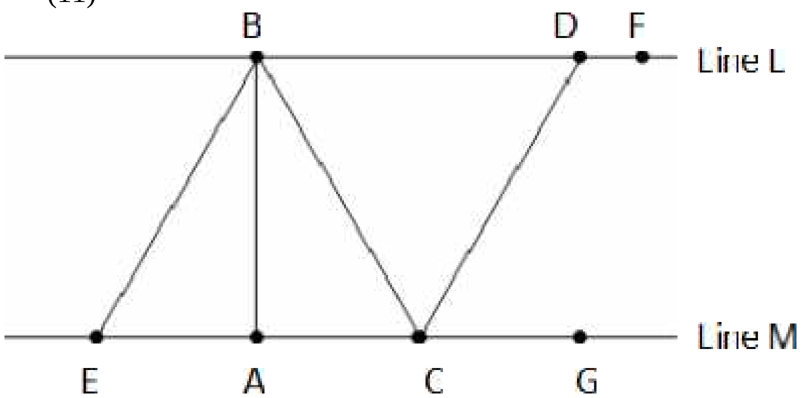
(9) What is the value of x?

(i) The average (arithmetic mean) of 5, x^2, 2, 10x, and 3 is -3.
(ii) The median of 109, -32, -30, 208, -15, x, 10, -43, 7 is -5.

(10) In 2003, a then-nascent Internet search engine developed an indexing algorithm called G-Cache that retrieved and stored X million webpages per hour. At the same time, a competitor developed an indexing algorithm called HTML-Compress that indexed and stored Y million pages per hour. If both algorithms indexed a positive number of pages per hour, was the number of pages indexed per hour by G-Cache greater than three times the number of pages indexed by HTML-Compress?

(i) On a per-hour basis in 2003, G-Cache indexed 1 million more pages than HTML-Compress indexed
(ii) HTML-Compress can index between 400,000 and 1.4 million pages per hour

(11)



If angle ABC is 30 degrees, what is the area of triangle BCE?

(i) Angle CDF is 120 degrees, lines L and M are parallel, and AC = 6, BC = 12, and EC = 2AC
(ii) Angle DCG is 60 degrees, angle CDG is 30 degrees, angle FDG = 90, and GC = 6, CD = 12 and EC = 12

(12) If both x and y are positive integers less than 100 and greater than 10, is the sum x + y a multiple of 11?

(i) x - y is a multiple of 22
(ii) The tens digit and the units digit of x are the same; the tens digit and the units digit of y are the same

(13) If b is prime and the symbol # represents one of the following operations: addition, subtraction, multiplication, or division, is the value of b # 2 even or odd?

(i) (b # 1) # 2 = 5
(ii) 4 # b = 3 # (1 # b) and b is even

(14) If x and y are both integers, which is larger, x^x or y^y?

(i) x = y + 1
(ii) $x^y > x$ and x is positive.

(15) A: $x^2 + 6x - 40 = 0$
B: $x^2 + kx + j = 0$

Which is larger, the sum of the roots of equation A or the sum of the roots of equation B?

(i) j = k
(ii) k is negative

(16) Given that A = 3y + 8x, B = 3y - 8x, C = 4y + 6x, and D = 4y - 6x, what is the value of x*y?

(i) AB + CD = -275
(ii) AD - BC = 420

(17) After a long career, John C. Walden is retiring. If there are 25 associates who contribute equally to a parting gift for John in an amount that is an integer, what is the total value of the parting gift?

(i) If four associates were fired for underperformance, the total value of the parting gift would have decreased by $200
(ii) The value of the parting gift is greater than $1,225 and less than $1,275

(18) If n and k are integers and $(-2)n^5 > 0$, is $k^{37} < 0$?

(i) $(nk)^z > 0$, where z is an integer that is not divisible by two
(ii) k < n

(19) What is the area of isosceles triangle X?

(i) The length of the side opposite the single largest angle in the triangle is 6cm
(ii) The perimeter of triangle X is 16cm

(20) For a set of 3 numbers, assuming there is only one mode, does the mode equal the range?

(i) The median equals the range
(ii) The largest number is twice the value of the smallest number

(21) Q is less than 10. Is Q a prime number?

(i) $Q^2 - 2 = P$; P is prime and P < 10.
(ii) Q + 2 is NOT prime, but Q is a positive integer.

(22) John is trying to get from point A to point C, which is 15 miles away directly to the northeast; however the direct road from A to C is blocked and John must take a detour. John must travel due north to point B and then drive due east to point C. How many more miles will John travel due to the detour than if he had traveled the direct 15 mile route from A to C?

(i) Tha ratio of the distance going north to the distance going east is 4 to 3
(ii) The distance traveled north going the direct route is 12

(23) If the product of X and Y is a positive number, is the sum of X and Y a negative number?

(i) $X > Y^5$
(ii) $X > Y^6$

(24) If x is a positive integer, is x divided by 5 an odd integer?

(i) x contains only odd factors
(ii) x is a multiple of 5

(25) Is $(2^{y+z})(3^x)(5^y)(7^z) < (90^y)(14^z)$?
(i) y and z are positive integers; x = 1
(ii) x and z are positive integers; y = 1

(26) If x is not zero, is $x^2 + 2x > x^2 + x$?
(i) $x^{\text{odd integer}} > x^{\text{even integer}}$
(ii) $x^2 + x - 12 = 0$

(27) How many prime numbers are there between the integers 7 and X, not-inclusive?

(i) $15 < X < 34$
(ii) X is a multiple of 11 whose sum of digits is between 1 and 7

(28) As a result of dramatic changes in the global currency market, the value of every item in Country X plummeted by 50% from 1990 to 1995. What was the value of a copy of St. Augustine's *Confessions* in Country X's currency in 1990? (Assume that the only variable influencing changes in the value of the book is the value of Country X's currency.)

(i) The value of St. Augustine's Confessions at the end of 1993 was $30
(ii) If the value of every item in Country X had plummeted by 50% from 1995 to 2000, the value of St. Augustine's *Confessions* in 2000 would have been $25

(29) If $10x + 10y + 16x^2 + 25y^2 = 10 + Z$, what is the value of x + y?

(i) $Z = (4x)^2 + (5y)^2$
(ii) x = 1

(30) Is $x|x|^3 < (|x|)^x$?

(i) $x^2 + 4x + 4 = 0$
(ii) $x < 0$

(31) If X is a positive integer, is X divisible by 4?

(i) X has at least two 2s in its prime factorization
(ii) X is divisible by 2

(32) Chef Martha is preparing a pie for a friend's birthday. How much more of substance X does she need than substance Y?

(i) Martha needs 10 cups of substance X
(ii) Martha needs the substances W, X, Y, and Z in the ratio: 15:5:2:1 and she needs 4 cups of substance Y

(33) How many computers did Michael, a salesman for the computer company Digital Electronics Labs, sell this past year that had more than 4GB of RAM and the Microsoft Windows Vista operating system? (Michael sold no computers with exactly 4GB of RAM)

(i) 40% of the 200 total computers that Michael sold had Vista and less than 4GB of RAM; these computers represent 80% of the total computers that Michael sold with Vista.
(ii) 50% of the 200 total computers that Michael sold had Vista; Of the computers that Michael sold without Vista, half had more than 4GB of ram while the other half had less than 4GB of RAM.

(34) x is a positive integer; is x + 17,283 odd?

(i) x - 192,489,358,935 is odd
(ii) x/4 is not an even integer

(35) If n is a positive integer, is n + 2 > z?

(i) $z^2 > n$
(ii) z – n < 0

(36) Peter can drive to work via the expressway or via the backroads, which is a less delay-prone route to work. What is the difference in the time Peter would spend driving to work via the expressway versus the backroads?

(i) Peter always drives 60mph, regardless of which route he takes; it takes Peter an hour to drive round-trip to and from work using the backroads
(ii) If Peter travels to and from work on the expressway, he spends a total of 2/3 of an hour traveling

(37) How many integers, x, satisfy the inequality b < x < a?

(i) a – b = 78
(ii) a > 100 and b < 50

(38) a, b, c, and d are integers; abcd≠0; what is the value of cd?

(i) c/b = 2/d
(ii) $b^3a^4c = 27a^4c$

(39) A cake recipe calls for sugar and flour in the ratio of 2 cups to 1 cup, respectively. If sugar and flour are the only ingredients in the recipe, how many cups of sugar are used when making a cake?

(i) the cake requires 33 cups of ingredients
(ii) the ratio of the number of cups of flour to the total number of cups used in the recipe is 1:3

(40) How many members of the staff of Advanced Computer Technology Consulting are women from outside the United States?

(i) one-fourth of the staff at Advanced Computer Technology Consulting are men
(ii) 20% of the staff, or 20 individuals, are men from the U.S.; there are twice as many women from the U.S. as men from the U.S.

(41) If x and y are integers, what is the ratio of 2x to y?

(i) $8x^3 = 27y^3$
(ii) $4x^2 = 9y^2$

(42) X, Y, and Z are three points in space; is Y the midpoint of XZ?

(i) ZY and YX have the same length
(ii) XZ is the diameter of a circle with center Y

(43) 15a + 6b = 30, what is the value of a-b?

(i) b = 5 – 2.5a
(ii) 9b = 9a – 81

(44) What is the value of $(n + 1)^2$?

(i) $n^2 - 6n = -9$
(ii) $(n-1)^2 = n^2 – 5$

(45) What is the remainder of a positive integer N when it is divided by 2?

(i) N contains odd numbers as factors
(ii) N is a multiple of 15

(46) X and Y are both positive integers whose combined factors include 3 and 7. Is the sum X + Y + 1 an odd integer?

(i) Both X and Y are divisible by 2
(ii) X + 2 = Y

(47) What is the average (arithmetic mean) of w, x, y, z, and 10?

(i) the average (arithmetic mean) of w and y is 7.5; the average (arithmetic mean) of x and z is 2.5
(ii) -[-z - y -x - w] = 20

(48) Is 13N a positive number?

(i) -21N is a negative number
(ii) $N^2 < 1$

(49) In triangle ABC, what is the measurement of angle C?

(i) The sum of the measurement of angles A and C is 120
(ii) The sum of the measurement of angles A and B is 80

(50) Police suspected that motorists on a stretch of I-75 often exceeded the speed limit yet avoided being caught through the use of radar detectors and jammers. Officer Johnson of the State Police recently pulled over a driver on I-75 and accused him of breaking the 50 mile-per-hour speed limit. Is Officer Johnson’s assertion correct?

(i) Officer Johnson noted that the driver had traveled 30 miles from point A to point B on I-75.
(ii) Officer Johnson noted that it took the driver 30 minutes to travel from point A to point B on I-75.

(51) n is a positive number; z – 15 is also a positive number; is z/n less than one?

(i) z – n > 0

(ii) n < 15

(52) Is (-x) a negative number?

(i) $4x^2 - 8x > (2x)^2 - 7x$
(ii) x + 2 > 0

(53) If A and B are integers, is B > A?

(i) B > 10
(ii) A < 10

(54) What is the value of xn – ny – nz?

(i) x – y – z = 10
(ii) n = 5

(55) If X is a positive integer, is X a prime number?

(i) X is an even number
(ii) 1 < X < 4

(56) Does x = y?
(i) x2 - y2 = 0
(ii) (x - y)2 = 0

(57) If R is an integer, is R evenly divisible by 3?
(i) 2R is evenly divisible by 3
(ii) 3R is evenly divisible by 3

(58) If he did not stop along the way, what speed did Bill average on his 3-hour trip?
(i) He travelled a total of 120 miles.
(ii) He travelled half the distance at 30 miles per hour, and half the distance at 60 miles per hour.

(59) Is x + y positive?
(i) x - y is positive.
(ii) y - x is negative.

(60) A shopper bought a tie and a belt during a sale. Which item did he buy at the greater dollar value?
(i) He bought the tie at a 20 percent discount.
(ii) He bought the belt at a 25 percent discount

1. Option A

(i) Evaluate statement (1) alone

a) Substitute the value of x from Statement (1) into the equation and manipulate it algebraically.

$$\sqrt[3]{x+y^2} = \sqrt[3]{y^2(y-1)+y^2}$$
$$= \sqrt[3]{y^3-y^2+y^2} = \sqrt[3]{y^3} = y$$

b) Since the question says that y is a positive integer, you know that the cube root of y^3, which equals y, will also be a positive integer. Statement (1) is SUFFICIENT.

(ii) Evaluate Statement (1) alone (Alternative Method).

a. For the cube root of a number to be an integer, that number must be an integer cubed. Consequently, the simplified version of this question is: "is $x + y^2$ equal to an integer cubed?"

b. Statement (1) can be re-arranged as follows:
$x = y^3 - y^2$
$y^3 = x + y^2$
Since y is an integer, the cube root of y^3, which equals y, will also be an integer.

c. Since $y^3 = x + y^2$, the cube root of $x + y^2$ will also be an integer. Therefore, the following will always be an integer:

$$\sqrt[3]{x+y^2}$$

d. Statement (1) alone is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a. Statement (2) provides minimal information. The question can be written as: "is the following cube root an integer?"

$$\sqrt[3]{2+y^2}$$

b. If y = 4, $x + y^2 = 2 + 4^2 = 18$ and the cube root of 18 is not an integer. However, if y = 5, $x + y^2 = 2 + 5^2 = 27$ and the cube root of 27 is an integer. Statement (2) is NOT SUFFICIENT.

Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

2. Option D

(i) In order for a number, *n*, to be divisible by 9, its digits must add to nine. Likewise, the remainder of the sum of the digits of *n* divided by 9 is the remainder when *n* is divided by 9. In other words:

$$\text{Remainder of } \frac{n}{9} = \text{Remainder of } \left[\frac{\text{Sum of Digits in n}}{9}\right]$$

(ii) To see this, consider a few examples:
Let $N = 901$
$901/9 = 100 + (R = 1)$
$(9+0+1)/9 = 10/9 = 1 + (R = 1)$

Let $N = 85$

85/9 = 9 + (*R* = *4*)
(8+5)/9 = 1 + (*R* = *4*)

Let *N* = 66
66/9 = 7 + (*R* = *3*)
(6+6)/9 = 1 + (*R* = *3*)

Let *N* = 8956
8956/9 = 995 + (*R* = *1*)
(8+9+5+6)/9 = 28/9 = 3 + (*R* = *1*)

(iii) Evaluate Statement (1) alone.

a. Based upon what was shown above, since the sum of the digits of *N* is always 13, we know that remainder of *N*/9 will always be the remainder of 13/9, which is R=4.
b. In case this is hard to believe, consider the following examples:
4540/9 = 504 + (*R* = *4*)
(4+5+4+0)/9 = 13/9 = 1 + (*R* = *4*)

1390/9 = 154 + (*R* = *4*)
(1+3+9+0)/9 = 13/9 = 1 + (*R* = *4*)

7231/9 = 803 + (*R* = *4*)
(7+2+3+1)/9 = 13/9 = 1 + (*R* = *4*)

1192/9 = 132 + (*R* = *4*)
(1+1+9+2)/9 = 13/9 = 1 + (*R* = *4*)

c. Statement (1) is SUFFICIENT.

(iv) Evaluate Statement (2) alone.

a. If adding 5 to a number makes it divisible by 9, there are 9-5=4 left over from the last clean division. In other words, *N*/9 will have a remainder of 4.
b. To help see this, consider the following examples:
Let *N* = 4
N+5=9 is divisible by 9 and *N*/9 -> R = 4

Let *N* = 13
N+5=18 is divisible by 9 and *N*/9 -> R = 4

Let *N* = 724
N+5=729 is divisible by 9 and *N*/9 -> R = 4

Let *N* = 418
N+5=423 is divisible by 9 and *N*/9 -> R = 4
c. Since *N* + 5 is divisible by 9, we know that the remainder of *N*/9 will always be 4. Statement (2) is SUFFICIENT.

Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

3. Option D

(i) Before even evaluating the statements, simplify the question. In a more complicated data sufficiency problem, it is likely that some rearranging of the terms will be necessary in order to see the correct answer.

(ii) Use the formula for a difference of squares $(a^2 - b^2) = (a + b)(a - b)$. However, let x^2 equal a, meaning $a^2 = x^4$.
$x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$

(iii) Recognize that the expression contains another difference of squares and can be simplified even further.
$(x^2 + y^2)(x^2 – y^2) = (x^2 + y^2)(x – y)(x + y)$

(iv) The question can now be simplified to: "If x and y are distinct positive integers, what is the value of $(x^2 + y^2)(x – y)(x + y)$?" If you can find the value of $(x^2 + y^2)(x - y)(x + y)$ or $x^4 - y^4$, you have sufficient data.

(v) Evaluate Statement (1) alone.

a) Statement (1) says $(y^2 + x^2)(y + x)(x - y) = 240$. The information in Statement (1) matches exactly the simplified question. Statement (1) is SUFFICIENT.

(vi) Evaluate Statement (2) alone.

a) Statement (2) says $x^y = y^x$ and $x > y$. In other words, the product of multiplying x together y times equals the product of multiplying y together x times.
b) The differences in the bases must compensate for the fact that y is being multiplied more times than x (since $x > y$ and y is being multiplied x times while x is being multiplied y times).
c) 4 and 2 are the only numbers that work because only 4 and 2 satisfy the equation $n^2= 2^n$, which is the condition that would be necessary for the equation to hold true.
d) Observe that this is true: $4^2 = 2^4 = 16$.
e) Remember that $x > y$, so $x = 4$ and $y = 2$. Consequently, you know the value of $x^4 - y^4$ from Statement (2). So, Statement (2) is SUFFICIENT.

Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

4. Option D

(i) In working on this question, it is helpful to remember that a number will be divisible by 9 if the sum of its digits equals 9.

(ii) Evaluate Statement (1) alone.

a. Based upon the information in Statement (1), it is helpful to plug in a few values and see if a pattern emerges:
$10^1 - 19 = -9$
$10^2 - 19 = 81$; the sum of the digits is 9, which is divisible by 9, meaning the entire expression is divisible by 9
$10^3 - 19 = 981$; the sum of the digits is $9 + 8 + 1=18$, which is divisible by 9, meaning the entire expression is divisible by 9
$10^4 - 19 = 9981$; the sum of the digits is $9(2) + 8 + 1=27$, which is divisible by 9, meaning the entire expression is divisible by 9
b. Notice that, in each instance, the sum of the digits is divisible by 9, meaning the entire expression is divisible by 9.
c. The pattern that emerges is that there are (n-2) 9s followed by the digit 8 and the digit 1.

d. The pattern of the sum of the digits of 10^n - 19 is 9(n-2) + 9 for all values of n > 1. (For n = 1, the sum is -9, which is also divisible by 9.) This means that the sum of the digits of 10^n - 19 is 9(n-1). Since this sum will always be divisible by 9, the entire expression (i.e., 10^n - 19) will always be divisible by 9.

e. Based upon this pattern, Statement (1) is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a. Statement (2) says that z + 981 is a multiple of 9. This can be translated into algebra:
9(a constant integer) = z + 981
Divide both sides by 9

$$\frac{z+981}{9} = \frac{z}{9} + \frac{981}{9} = \text{integer}$$

b. Since 981 is divisible by 9 (its digits sum to 18, which is divisible by 9), you can further rewrite Statement (2).

$$\frac{z+981}{9} = \frac{z}{9} + \text{integer} = \text{integer}$$

c. Since an integer minus an integer is an integer, Statement (2) can be rewritten even further. Since z divided by 9 is an integer, z is divisible by 9. Statement (2) is SUFFICIENT.

$$\frac{z}{9} = \text{integer} - \text{integer} = \text{integer}$$

Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

5. Option C

(i) Evaluate statement (1) alone

(a) Statement (1) says that the sum of any two factors is even. The sum of two integers is even under two circumstances:
odd + odd = even
even + even = even

(b) Since the sum of any two factors is even, all the factors must have the same parity. If x had both even and odd factors, then it would be possible for two factors to add together and be odd (remember that an odd number + an even number = an odd number and, in Statement 1, the sum of *any two positive factors* must be even).

(c) Since the problem says "the sum of any two positive factors of x is even" and the number 1 is a factor of any number, x must only contain odd factors. If x contained one even factor, it would be possible to add that even factor with the number one and the result would be an odd number. Since the number 2 is a factor of every even number, x cannot be even. Otherwise, it would be possible to add the factors 1 and 2 together and their sum would not be even.

(d) Statement (1), when inspected carefully, says that x is an odd number that only contains odd factors. Since there are many possibilities (x = 1, 3, 5, 7, 9, 11, 15, ...), Statement (1) is NOT SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a. Statement (2) says that x is a prime number less than 4. Remember that x must also be a positive integer (as per the original question). Although this narrows the possibilities for x, because there are still two possibilities (x = 2 or x = 3; both these values are prime, less than 4, and positive integers), Statement (2) is NOT SUFFICIENT. Please remember that the number one is not prime.

(iii) Evaluate Statements (1) and (2) together.

Statements (1) and (2), when taken together, definitively show that x = 3. Statements (1) and (2), when taken together, are SUFFICIENT. Answer choice C is correct.

6. Option A

(i) Simplify the question: since raising a number to an odd power does not change the sign, x is a positive integer.

(ii) The question, is w - z > $5(7^{x-1} - 5^x)$?, can be simplified to: is w - z > $5*7^{x-1} - 5^{x+1}$?

(iii) Evaluate Statement (1) alone.

(a) Statement (1) allows the following substitution:
Is 7^x - (a number less than 25) > $5(7^{x-1}) - 5^{x+1}$?
Equivalently: Is 7^x - (a number less than 5^2) > $5(7^{x-1}) - 5^{x+1}$?

(b) If this question can be answered definitively for all legal values of x (i.e., positive integers), Statement (1) is sufficient. Although this statement is difficult to evaluate algebraically, a little logic makes Statement (1) plainly sufficient. It is helpful to step back and see the logic about to be employed.

a - b will always be greater than c - d if these numbers are positive and a > c and b < d. In this situation, a smaller number (b is smaller than d) is being subtracted from a larger number (a is greater than c). Consequently, if the left side of the equation starts from a larger number and subtracts a smaller number than the right side of the equation, it is quite clear that the difference on the left side will be larger than the difference on the right side of the inequality.

For example: 10 - 2 > 5 - 4
You are starting with a larger number on the left (i.e., 10 > 5) and subtracting a smaller number on the left (2 < 4). Consequently, it only makes sense that the number on the left is going to be larger.

(c) This same logic holds true in the inequality derived in Statement (1). Since x is a positive integer (it is essential to know this), 7^x will be bigger than $5(7^{x-1})$. You know this is true because there will be x sevens on the left side of the inequality and (x-1) sevens on the right side of the inequality. The extra 7 on the left will out-weight the extra 5 on the right, making the left side start with a larger number.

(d) Continuing with this logic, (a number less than 5^2) will be less than 5^{x+1} since x is a positive integer and the smallest possible value for x (i.e., 1) makes $5^{x+1} = 5^{1+1} = 5^2 = 25$. Since 5^{x+1} will always be at least 25, it will always be greater than (a number less than 25). Statement (1) is SUFFICIENT.
Note: If z were a negative number, which it could be, the inequality would still hold true. It would make the left side of the inequality even larger as we would effectively be adding a number to 7^x.

(iv) Evaluate Statement (2) alone.

(a) Statement (2) says that x = 4. The inequality can now be re-written:
is w - z > $5(7^{4-1} - 5^{4+1})$?
In other words, is w - z > 1,715 - 3,125?
Or, to simplify it as much as possible:
is w - z > -1,410?
If w = 7^4 = 2,401 and z = 1, the answer is YES. However, if w = -100,000 (nothing in Statement (2) precludes this possibility—do not import information over from Statement (1)) and z = 1, the answer is NO. Since Statement (2) does not provide

enough information to definitively answer the original question, it is NOT SUFFICIENT.

Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

7. Option B

(i) In order to solve this question efficiently, it is necessary to begin with number properties. For a product of any number of terms to be odd, all the terms must be odd. If there is but one even term, the product will be even. To see this, consider the following examples:
All Terms Odd --> Odd Product
3*7*9*5 = 945
7*9*3 = 189
1*3*5 = 15

But: One or More Even Terms --> Even Product
3*7*9*2 = 378
7*9*4 = 252
1*3*5*6 = 90

(ii) In order for $(x^3 + 19837)(x^2 + 5)(x – 3)$ to be an odd number, all the terms must be odd.

(iii) To determine under what conditions each term will be odd, it is important to remember the following relationships:
odd + odd = even
odd - odd = even

even + even = even
even - even = even

even + odd = odd
even - odd = odd
odd + even = odd
odd - even = odd

(iv) The only way for each term of $(x^3 + 19837)(x^2 + 5)(x – 3)$ to be odd is if an even and an odd number are added or subtracted together within the parenthesis of each term. In other words:
even + odd = odd: For $(x^3 + 19837)$ to be odd, since 19837 is odd, x^3 will need to be even. This will happen only when x is even.

even + odd = odd: For $(x^2 + 5)$ to be odd, since 5 is odd, x^2 will need to be even. This will happen only when x is even.

even - odd = odd: For $(x – 3)$ to be odd, since 3 is odd, x will need to be even.

(v) When combining the results from the analysis of the three terms above, the only way for $(x^3 + 19837)(x^2 + 5)(x – 3)$ to be odd is if each term is odd. This will only happen if x is even. Consequently, the original question can be simplified to: is x even? Another version of the simplified question is: what is the parity of x?

(vi) Evaluate Statement (1) alone.

(a) In order for the sum of any prime factor of x and x to be even, it must follow one of two patterns:
Pattern (1): even + even = even
Pattern (2): odd + odd = even

(b) There are two possible cases:
Case (1): x is even. In this case, Pattern (1) must hold. Since x is even in Case (1), any and every prime factor of x must be even (otherwise we could choose an odd prime factor of x and the sum of x and the odd prime factor would be odd). Let's consider some examples:
Let x = 12: However, x cannot equal 12 since one prime factor of 12 is 3 and 12 + 3 = odd number.
Let x = 26: However, x cannot equal 26 since one prime factor of 26 is 13 and 26 + 13 = odd number.
Let x = 14: However, x cannot equal 14 since one prime factor of 14 is 7 and 14 + 7 = odd number.
Let x = 16: x can equal 16 since every prime factor of 16 is even and as a result we know that and 16 + any prime factor = even number.
It is clear that Statement (1) allows x to be even (e.g., 16 is a possible value of x).
Case (2): x is odd. In this case, Pattern (2) must hold. Since x is odd in Case (2), any and every prime factor of x must be odd (otherwise we could choose an even prime factor of x and the sum of x and the even prime factor would be odd). Since all the prime factors of x are odd, x must be odd in Case (2). Let's consider some examples:
Let x = 11: Every prime factor of 11 is odd, so: 11 + prime factor of 11 = even number.
Let x = 15: Every prime factor of 15 is odd, so: 15 + prime factor of 15 = even number.

(c) Since Statement (1) allows x to be either even (e.g., 16) or odd (e.g., 15), we cannot determine the parity of x.

(d) Statement (1) is NOT SUFFICIENT.

(vii) Evaluate Statement (2) alone.

(a) 3x = Even Number
(odd)(x)=(even)
x must be even because, as shown above, if x were odd, 3x would be odd.

Statement (2) is SUFFICIENT since it definitively tells the parity of x.

(b) Since Statement (1) alone is NOT SUFFICIENT but Statement (2) alone is SUFFICIENT, answer B is correct.

8. Option B

(i) For some students, the theoretical nature of this question makes it intimidating. For these individuals, we recommend picking numbers as a means of determining sufficiency.

(ii) Evaluate Statement (1) alone.

a. Draw a table to quickly pick numbers in order to determine whether Statement (1) is sufficient. It is quickest to choose numbers for D+1 and C+1 that work (i.e., produce a remainder of 5) and then infer the values of D and C.
Let R(X/Y) = the remainder of X/Y

D	C	D+1	C+1	R[(D+1)/(C+1)]	R(D/C)
14	9	15	10	5	5
22	5	23	6	5	2
44	19	45	20	5	6

b. Different legitimate values of D+1 and C+1 yield different remainders for D/C. Consequently, the information in Statement (1) is not sufficient to determine the remainder when D is divided by C.

c. Algebraically, we know that D+1 divided by C+1 will not have the same remainder as D divided by C since fractions do not stay equivalent when you add to them (i.e., x divided by y does not equal x+1 divided by y+1).
d. Statement (1) alone is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a. Before evaluating Statement (2), it is essential to simplify by factoring the numerator:
ND + NC = N(D+C)
Cancel out the N in both the numerator and denominator. Statement (2) can be simplified to: If D+C is divided by C, the remainder is 5.
b. We can further simplify by noticing that D+C divided by C is equal to D divided by C plus C divided by C.
$$\frac{D+C}{C} = \frac{D}{C} + \frac{C}{C} = \frac{D}{C} + 1$$
c. There are two parts to this equation: (1) D divided by C (2) the number 1
The sum of parts (1) and (2) will always have a remainder of 5 (this is what Statement 2 says). This remainder cannot come from the second part (i.e., C divided by C equals *+1* and there is no remainder).
Consequently, the remainder of 5 must come from D divided by C. So, we know that D divided by C will always produce a remainder of 5, which provides sufficient information to answer the original question.
d. Statement (2) alone is SUFFICIENT.

(iv) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

9. Option D
(i) Evaluate statement (1) alone
(a) Based upon the formula for the average, you know that:
$(5 + x^2 + 2 + 10x + 3)/5 = -3$
$x^2 + 10x + 5 + 2 + 3 = -15$
$x^2 + 10x + 5 + 2 + 3 + 15 = 0$
$x^2 + 10x + 25 = 0$
$(x + 5)^2 = 0$
$x = -5$
(b) Statement (1) alone is SUFFICIENT.

(ii) Evaluate Statement (2) alone.

(a) Order the numbers in ascending order without x:
-43, -32, -30, -15, 10, 7, 109, 208
(b) Consider the possible placements for x and whether these would make the median equal to -5:
Case (1): x, -43, -32, -30, -15, 10, 7, 109, 208
Median: -15
Not a possible case since the median is not -5.

Case (2): -43, x, -32, -30, -15, 10, 7, 109, 208
Median: -15
Not a possible case since the median is not -5.

Case (3): -43, -32, x, -30, -15, 10, 7, 109, 208
Median: -15

Not a possible case since the median is not -5.

Case (4): -43, -32, -30, x, -15, 10, 7, 109, 208
Median: -15
Not a possible case since the median is not -5.

Case (5): -43, -32, -30, -15, x, 10, 7, 109, 208
Median: x
A possible case since the median is x, which can legally be -5.
In this case, x must be -5 in order for the median of the set to be -5, which must be according to Statement (2).

Case (6): -43, -32, -30, -15, 10, x, 7, 109, 208
Median: 10
Not a possible case since the median is not -5.

Case (7): -43, -32, -30, -15, 10, 7, x, 109, 208
Median: 10
Not a possible case since the median is not -5.

Case (8): -43, -32, -30, -15, 10, 7, 109, x, 208
Median: 10
Not a possible case since the median is not -5.

Case (9): -43, -32, -30, -15, 10, 7, 109, 208, x
Median: 10
Not a possible case since the median is not -5.

(c) Since Statement (2) tells us that the median must be -5, we know that x must be a value such that the median is -5. This can only happen in Case 5. Specifically, it can only happen when x = -5. Since the median must equal -5 and this can only happen when x = -5, we know that x = -5.

(d) Statement (2) alone is SUFFICIENT.

(iii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

10. Option E

(i) Translate the final sentence, which contains the question, into algebra:
"the number of pages indexed per hour by G-Cache" = X
"greater than three times" translates into: >3
"the number of pages indexed by HTML-Compress" = Y

Putting this together:
Was X > 3Y?

(ii) Evaluate Statement (1) alone.

a. Translate the information from Statement (1) into algebra:
X - Y = 1 million
b. Since the original question states that "both algorithms indexed a positive number of pages per hour", the following inequalities must hold true:
X > 0
Y > 0

c. Simply knowing that X - Y = 1 million does not provide enough information to determine whether X > 3Y.
This can be seen via an algebraic substitution or by trying different numbers.
d. Trying Numbers
Let X = 10 and, therefore, Y = 9
10 is NOT > 3(9)
But, let X = 1.1 and, therefore, Y = .1
1.1 IS > 3(.1)
e. Algebraic Substitution
X - Y = 1 million
X = Y + 1 million
Plug this into the inequality we are trying to solve for:
Was X > 3Y?
Was (Y + 1 million) > 3Y?
Was 1 million > 2Y?
Was 500,000 > Y?
Was Y < 500,000?

Simply knowing that X - Y = 1 million does not provide enough information to determine whether Y < 500,000

f. Since different legitimate values of Y produce different answers to the question of whether X > 3Y, Statement (1) is not sufficient.
g. Statement (1) is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a. Translate the information from Statement (2) into algebra:
400,000 < Y < 1,400,000
b. We know nothing about the value of X.
If X were 10 million, the answer to the original question was X > 3Y? would be "yes."
If X were 100,000, the answer to the original question was X > 3Y? would be "no."
c. Since different legitimate values of X and Y produce different answers to the question of whether X > 3Y, Statement (2) is not sufficient.
d. Statement (2) is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2) together.

a. With the information in Statement (1), we concluded that the original question can be boiled down to:
Is Y < 500,000?
b. Statement (2) says:
400,000 < Y < 1,400,000
c. Even when combining Statements (1) and (2), we cannot determine whether Y < 500,000
Y could be 450,000 (in which case X = 1,450,000) or Y could be 650,000 (in which case X = 1,650,000). These two different possible values of X and Y would produce different answers to the question "Was Y < 500,000?" Consequently, we would have different answers to the question "Was X > 3Y?"
d. Statements (1) and (2), even when taken together, are NOT SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT, Statement (2) alone is NOT SUFFICIENT, and Statements (1) and (2), even when taken together, are NOT SUFFICIENT, answer E is correct.

11. Option D
(i) Even though lines L and M look parallel and angle BAC looks like a right angle, you cannot make these assumptions.
(ii) The formula for the area of a triangle is .5bh

(iii) Evaluate Statement (1) alone.

(a) Since EC = 2AC, EA = CA, EC = 2(6) = 12 and line AB is an angle bisector of angle EBC. This means that angle ABC = angle ABE. Since we know that angle ABC = 30, we know that angle ABE = 30. Further, since lines L and M are parallel, we know that line AB is perpendicular to line EC, meaning angle BAC is 90.
(b) Since all the interior angles of a triangle must sum to 180:
angle ABC + angle BCA + angle BAC = 180
30 + angle BCA + 90 = 180
angle BCA = 60
(c) Since all the interior angles of a triangle must sum to 180:
angle BCA + angle ABC + angle ABE + angle AEB = 180
60 + 30 + 30 + angle AEB = 180
angle AEB = 60
(d) This means that triangle BCA is an equilateral triangle.
(e) To find the area of triangle BCE, we need the base (= 12 from above) and the height, i.e., line AB. Since we know BC and AC and triangle ABC is a right triangle, we can use the Pythagorean theorem on triangle ABC to find the length of AB.
$6^2 + (AB)^2 = 12^2$
$AB^2 = 144 - 36 = 108$
$AB = 108^{1/2}$
(f) Area = .5bh
Area = $.5(12)(108^{1/2}) = 6*108^{1/2}$
(g) Statement (1) is SUFFICIENT

(iv) Evaluate Statement (2) alone.

(a) The sum of the interior angles of any triangle must be 180 degrees.
DCG + GDC + CGD = 180
60 + 30 + CGD = 180
CGD = 90
Triangle CGD is a right triangle.
(b) Using the Pythagorean theorem, DG = $108^{1/2}$
$(CG)^2 + (DG)^2 = (CD)^2$
$6^2 + (DG)^2 = 12^2$
DG = $108^{1/2}$
(c) At this point, it may be tempting to use DG = $108^{1/2}$ as the height of the triangle BCE, assuming that lines AB and DG are parallel and therefore AB = $108^{1/2}$ is the height of triangle BCE. However, we must show two things before we can use AB = $108^{1/2}$ as the height of triangle BCE: (1) lines L and M are parallel and (2) AB is the height of triangle BCE (i.e., angle BAC is 90 degrees).
(d) Lines L and M must be parallel since angles FDG and CGD are equal and these two angles are alternate interior angles formed by cutting two lines with a transversal. If two alternate interior angles are equal, we know that the two lines that form the angles (lines L and M) when cut by a transversal (line DG) must be parallel.
(e) Since lines L and M are parallel, DG = the height of triangle BCE = $108^{1/2}$. Note that it is not essential to know whether AB is the height of triangle BCE. It is sufficient to know that the height is $108^{1/2}$. To reiterate, we know that the height is 8 since the height of BCE is parallel to line DG, which is $108^{1/2}$.

(f) Since we know both the height ($108^{1/2}$) and the base (CE = 12) of triangle BCE, we know that the area is: $.5*12*108^{1/2} = 6*108^{1/2}$
(g) Statement (2) alone is SUFFICIENT.

(v) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

12. Option B
(i) If both x and y are multiples of 11, then both x + y and x - y will be multiples of 11. In other words, if two numbers have a common divisor, their sum and difference retain that divisor. In case this is hard to conceptualize, consider the following examples:
42 - 18 {both numbers share a common factor of 6}
=(6*7) - (6*3)
=6(7 - 3)
=6(4)
=24 {which is a multiple of 6}

49 + 14 {both numbers share a common factor of 7}
=(7*7) + (7*2)
=7(7+2)
=7*9
=63 {which is a multiple of 7}
(ii) However, if x and y are not both multiples of 11, it is possible that x - y is a multiple of 11 while x + y is not a multiple of 11. For example:
68 - 46 = 22 but 68 + 46 = 114, which is not divisible by 11.
The reason x - y is a multiple of 11 but not x + y is that, in this case, x and y are not individually multiples of 11.

(iii) Evaluate Statement (1) alone.

(a) Since x-y is a multiple of 22, x-y is a multiple of 11 and of 2 because 22=11*2
(b) If both x and y are multiples of 11, the sum x + y will also be a multiple of 11. Consider the following examples:
44 - 22 = 22 {which is a multiple of 11 and of 22}
44 + 22 = 66 {which is a multiple of 11 and of 22}
88 - 66 = 22 {which is a multiple of 11 and of 22}
88 + 66 = 154 {which is a multiple of 11 and of 22}
(c) However, if x and y are not individually divisible by 11, it is possible that x - y is a multiple of 22 (and 11) while x + y is not a multiple of 11. For example:
78 - 56 = 22 but 78 + 56 = 134 is not a multiple of 11.
(d) Statement (1) alone is NOT SUFFICIENT.

(iv) Evaluate Statement (2) alone.

(a) Since the tens digit and the units digit of x are the same, the range of possible values for x includes:
11, 22, 33, 44, 55, 66, 77, 88, 99
Since each of these values is a multiple of 11, x must be a multiple of 11.
(b) Since the tens digit and the units digit of y are the same, the range of possible values for y includes:
11, 22, 33, 44, 55, 66, 77, 88, 99
Since each of these values is a multiple of 11, y must be a multiple of 11.
(c) As demonstrated above, if both x and y are a multiple of 11, we know that both x + y and x - y will be a multiple of 11.
(d) Statement (2) alone is SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

13. Option D

(i) This problem deals with the properties of prime numbers. Keep in mind that 1 is not a prime number and that 2 is the only even prime number.

(ii) Evaluate Statement (1) alone.

a. Try each of the operations in turn. First, try addition:
(b + 1) + 2 = 5
Solve for b.
b = 2.
Under addition, b = 2, which is a prime number; therefore addition is a possibility for the operator.
b. Next, try subtraction.
(b - 1) - 2 = 5
Solve for b.
b = 8
But b = 8 is not prime, therefore operator cannot represent subtraction.
c. Next, try multiplication.
(b * 1) * 2 = 5
Solve for b.
b = 5/2
But b = 5/2 is not prime, therefore operator cannot represent multiplication.
d. Finally, try division.
(b / 1) / 2 = 5
Solve for b.
b = 10
But b = 10 is not prime, therefore operator cannot represent division.
e. Since addition is the only operation for which b is prime, # must represent addition. In this case, b = 2 and the value of b # 2 is 4, which is even.
f. Statement (1) is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a. Try each of the operations in turn. First, try addition:
4 + b = 3 + (1 + b)
Subtract 4 from each side.
b = b
While this is true, it does not give any information about the value of b. However, addition is still a possible operation.
b. Next, try subtraction:
4 - b = 3 - (1 - b)
Solve for b.
b = 1
In this case, b = 1 is not a prime number, so subtraction is not a possible operation.
c. Next, try multiplication:
4 * b = 3 * (1 * b)
Simplify.
4b = 3b
The only value for which this holds true is b = 0, which is not a prime number. Therefore, multiplication is not a possible operation.

d. Finally, try division:
$4 / b = 3 / (1 / b)$
Multiply both sides by $(1 / b)$
$4 / b^2 = 3$
Solve for b^2.
$b^2 = 4/3$
Which means that b = sqrt(4/3) or b = -sqrt(4/3). Neither of these is prime, so division is not a possible operation.

e. The symbol # must represent addition, since this is the only possible operation. By Statement (2), b is even, but b is still prime. Since 2 is the only even prime number, b must be 2. In this case, the value of b # 2 is even because an even number plus 2 is still an even number.

f. Statement (2) is SUFFICIENT.

(iv) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

14. Option C

(i) The problem deals with properties of exponents. Analyzing the different cases where x is positive and y is positive, for example, is the key to this problem.

(ii) Evaluate Statement (1) alone.

a. Since $x = y + 1$, substitute for x in x^x.
$x^x = (y + 1)^{(y + 1)}$

b. Since x is one number larger than y, it may appear that x^x must be larger than y^y. However, consider the table below.

x	y	x^x	y^y
-1	-2	-1	1/4
-2	-3	1/4	-1/27
-3	-4	-1/27	1/256

c. When x = -1 and y = -2, x^x is smaller. However, when x = -2 and y = -3, y^y is smaller. Whether x^x or y^y is larger depends on the values of x and y.

d. Statement (1) is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a. Given the inequality from Statement (2),
$x^y > x$
Divide both sides by x.
$x^{(y - 1)} > 1$

b. First consider this inequality when y = 1. Then $x^{(y - 1)} = x^{(1 - 1)} = 1$. But this violates the inequality because it is not true that $x^{(1 - 1)} > 1$. Therefore, y may not be 1.

c. Next consider the case where y < 1. Then $x^{(y - 1)} = x^{-k}$, where -k is some negative number. And $x^{-k} = 1 / x^k$, which is less than 1 no matter the value of x; this violates the inequality, too, since $x^{(y - 1)}$ is supposed to be greater than 1. For example, if y = -3 and x = 2, then $x^{(y - 1)} = 2^{(-3 - 1)} = 1 / 2^4 = 1/8$, which is less than 1.

d. Since it cannot be that y = 1 or y < 1, the only option that remains is y > 1. From this conclusion and the information given in Statement (2), we conclude that x > 0 and y > 1. However, this is not enough information to determine whether x^x or y^y is larger. For example, it could be that x = 4 and y = 6; in this case, y^y would be larger. It could be that x = 7 and y = 3; in this case, x^x would be larger.

e. Statement (2) is NOT SUFFICIENT.

(iv) Evaluate Statement (1) and (2) together.

a. The conclusion reached in examining Statement (2) was that $y > 1$ and $x > 0$. Combine this with Statement (1), which says that x is one number larger than y. Thus, x^x will always be larger than y^y. For example, if $y = 2$, then $x = 3$; $y^y = 2^2 = 4$ and $x^x = 3^3 = 27$.
b. Statement (1) and (2) together are SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT yet Statements (1) and (2), when taken together, are SUFFICIENT, answer C is correct.

15. Option B

(i) This problem combines the quadratic formula with properties of positive and negative numbers. First, find the sum of the roots in equation A using the quadratic formula or factoring.
Using the quadratic formula:
x = (-6 + sqrt(36 - 4(1)(-40))) / 2 and (-6 - sqrt(36 - 4(1)(-40))) / 2 are the roots.

Using factoring:
$x^2 + 6x - 40 = 0$
$(x + 10)(x - 4) = 0$
$x = -10, 4$

(ii) To find the sum, these two roots will be added. Notice that one root contains +sqrt(36 + 160) and the other contains -sqrt(36 + 160). When these two terms are added, they equal zero. Thus, the only terms left in the sum are -6/2 and -6/2. Add these together to find the sum of the roots: -6/2 + (-6/2) = -6. Notice that the sum of the roots equals -b, where b is the coefficient of the x term.

(iii) In fact, in any sum of quadratic roots, the +sqrt(...) and -sqrt(...) terms will cancel. Therefore, for any quadratic equation the sum of the roots is -b, where b is the coefficient of the x term ($ax^2 + bx + c = 0$). This fact will simplify the problem greatly.

(iv) Evaluate Statement (1) alone.

a. The sum of the roots for equation A was found to be -6. Using the fact demonstrated above, the sum of the roots of equation B is -k. Statement (1) says that that j = k, which means that the sum of the roots of equation B is -k = -j.
b. However, nothing is known about j and k. It could be that j = -7, in which case the sum of the roots of B is -(-7) = 7, which is larger than the sum of the roots of A. However, it could be that j = 9, in which case the sum of the roots of B is -(9) = -9, which is smaller than the sum of the roots of A. It cannot be determined which sum is larger.
c. Note: We cannot assume that j and k are integers as the problem does not state this. If we knew they were integers, then j = k = 2 since this is the only way for j to equal k in $x^2 + jx + k = 0$, and we could solve the problem.
d. Statement (1) is NOT SUFFICIENT.

(v) Evaluate Statement (2) alone.

a. If k is negative, then the sum of the roots of B is -k, which is the negative of a negative number, making the sum positive. And since this sum is positive, it is larger than the sum of the roots of A, which is -6.
b. Statement (2) is SUFFICIENT.

(vi) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

16. Option B

(i) This problem deals with polynomials and factoring, as well as simultaneous equations with two variables. Factoring or expanding where necessary will help greatly in solving this problem.

(ii) Evaluate Statement (1) alone.

a. First, multiply A and B, then C and D.
   $AB = (3y + 8x)(3y - 8x) = 9y^2 - 64x^2$
   $CD = (4y + 6x)(4y - 6x) = 16y^2 - 36x^2$
b. Now add AB and CD.
   $AB + CD = 25y^2 - 100x^2$
   Factor a 25 out of the right side of the equation.
   $AB + CD = 25(y^2 - 4x^2)$
   Notice that the polynomial on the right side can be factored.
   $AB + CD = 25(y + 2x)(y - 2x)$
c. Since AB + CD = -275, substitute this value into the equation.
   $-275 = 25(y + 2x)(y - 2x)$
   Divide both sides by 25.
   $-11 = (y + 2x)(y - 2x)$
d. Let P = y + 2x and Q = y - 2x. There are only four ways that -11 can be the product of the two numbers P and Q: P = -1 and Q = 11, or P = -11 and Q = 1, or P = 1 and Q = -11, or P = 11 and Q = -1. Examine the first two possibilities.
e. First, P = -1 and Q = 11. Write out P and Q fully.
   $P = y + 2x = -1$
   $Q = y - 2x = 11$
   Using linear combination, add both sides of the two equations together.
   $2y = 10$
   Which means that y = 5. Plug y = 5 back into either equation and get x = -3.
f. Secondly, P = -11 and Q = 1. Write out P and Q fully.
   $P = y + 2x = -11$
   $Q = y - 2x = 1$
   Using linear combination, add both sides of the two equations together.
   $2y = -10$
   Which means that y = -5. Plug y = -5 back into either equation and get x = -3.
g. In the first case, y = 5 and x = -3, which means that x*y = -15. However, in the second case, when y = -5 and x = -3, x*y = +15. Therefore it is not possible to determine the value of x*y since the sign cannot be determined.
h. Statement (1) is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a. First, multiply A and D, then B and C.
   $AD = (3y + 8x)(4y - 6x) = 12y^2 + 14xy - 48x^2$
   $BC = (3y - 8x)(4y + 6x) = 12y^2 - 14xy - 48x^2$
b. Now subtract BC from AD; almost all the terms cancel out.
   $AD - BC = 14xy - (-14xy) = 28xy$
   Since AD - BC = 420, substitute this value into the equation.
   $420 = 28xy$.
   Divide both sides by 28.
   $xy = 15$

c. Statement (2) is SUFFICIENT.

(iv) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

17. Option D

(i) Simplify the question by translating it into algebra.
Let P = the total value of John's parting gift
Let E = the amount each associate contributed
Let N = the number of associates
P = NE = 25E

(ii) With this algebraic equation, if you find the value of either P or E, you will know the total value of the parting gift.

(iii) Evaluate Statement (1) alone.

Two common ways to evaluate Statement (1) alone:

(iv) Statement 1: Method 1

a) Since the question stated that each person contributed equally, if losing four associates decreased the total value of the parting gift by $200, then the value of each associate's contribution was $50 (=$200/4).
b) Consequently, P = 25E = 25(50) = $1,250.

(v) Statement 1: Method 2

a) If four associates leave, there are N - 4 = 25 - 4 = 21 associates.
b) If the value of the parting gift decreases by $200, its new value will be P - 200.
c) Taken together, Statement (1) can be translated:
P - 200 = 21E
P = 21E + 200
d) You now have two unique equations and two variables, which means that Statement (1) is SUFFICIENT.
e) Although you should not spend time finding the solution on the test, here is the solution.
Equation 1: P = 21E + 200
Equation 2: P = 25E
P = P
25E = 21E + 200
4E = 200
E = $50
f) P = NE = 25E = 25($50) = $1250

(vi) Evaluate Statement (2) alone.

a) Statement (2) says that $1,225 < P < $1,275. It is crucial to remember that the question stated that "25 associates contribute equally to a parting gift for John in an amount that is an integer." In other words P / 25 must be an integer. Stated differently, P must be a multiple of 25.
b) There is only one multiple of 25 between 1,225 and 1,275. That number is $1,250. Since there is only one possible value for P, Statement (2) is SUFFICIENT.

(vii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

18. Option D

(i) It is important to begin by simplifying the question.
Since k is raised to an odd power, k^{37} will always be less than 0 if k is less than 0. Likewise, k^{37} will always be greater than 0 if k is greater than 0.
So, the question can be simplified to: is $k < 0$?
$k^{(\text{odd integer})} < 0$ if $k < 0$
$k^{(\text{odd integer})} > 0$ if $k > 0$

(ii) The question can be simplified even more. Since (-2)(negative number) > 0 and (-2)(positive number) < 0, you know n^5 is a negative number. This means that $n < 0$. If n were greater than 0, the statement $(-2)n^5 > 0$ would never be true.

(iii) Summarizing in algebra:
(-2)(negative number) > 0
(-2)(positive number) < 0
$(-2)(n^5) > 0$
$n^5 < 0$
Therefore: $n < 0$

(iv) The fully simplified question is: "if n and k are integers and $n < 0$, is $k < 0$?"

(v) Evaluate Statement (1) alone.

a) By saying that "z is an integer that is not divisible by 2," Statement (1) is saying that z is an odd integer. So, any base raised to z will keep its sign (i.e., whether the expression is positive or negative will not change since the base is raised to an odd exponent).
$z/2$ = not integer if z is odd
$z/2$ = integer if z is even

b) Remember that $(nk)^z = (n^z)(k^z)$. So, Statement (1) says that $(n^z)(k^z) > 0$. It is important to know that there are two ways that a product of two numbers can be greater than zero:
Case 1: (negative number)(negative number) > 0
Case 2: (positive number)(positive number) > 0

c) Since you know that $n < 0$, we are dealing with Case 1 and Statement (1) can be simplified even further:
(negative number)($k^{(\text{odd exponent})}$) > 0.

d) Since k will not change its sign when raised to an odd exponent, the equation can be simplified even further:
(negative number)(k) > 0. k must be a negative number. Otherwise, this inequality will not be true.

e) To summarize in algebra:
$(nk)^z > 0$
$(nk)^z = (n^z)(k^z)$
$(n^z)(k^z) > 0$
(negative number)(negative number) > 0
or (positive number)(positive number) > 0
(negative number)($k^{(\text{odd exponent})}$) > 0
(negative number)(k) > 0
k is negative

f) Since k is a negative number, $k^{37} < 0$. Statement (1) is SUFFICIENT.

(vi) Evaluate Statement (2) alone.

a) Statement (2) says that k is less than n. Since you know that n is less than 0, Statement (2) says that k is less than a negative number. Only a negative number is less than another negative number. So, k must also be a negative number.

Consequently, k^{37} will always be less than 0 since $(\text{negative})^{odd} < 0$. Statement (2) is SUFFICIENT.

b) Summarizing in algebra:
$k < n < 0$

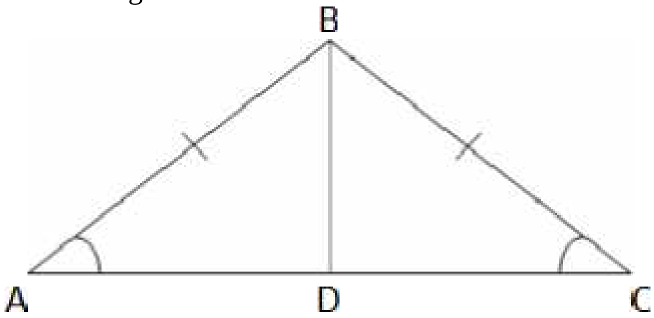
(vii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

19. Option C

(i) In a triangle, the side opposite the largest angle will be the longest. Correspondingly, the side opposite the smallest angle will be the shortest.

(ii) Evaluate Statement (1) alone.

a. The side opposite the single largest angle must be the single largest side in the triangle. Since an isosceles triangle contains two equal sides and two equal angles yet we know that a "single largest angle" exists, the side opposite the *single* largest angle cannot be one of these equal sides. If the longest side of an isosceles triangle were one of its equal sides, both angles opposite the equal sides would have equal measurement and there would be no *single* largest angle as Statement (1) indicates there must be.
b. The single largest side must be the base since the two other angles will have equivalent measurements and thus the length of the sides opposite them will be equivalent. The two equal sides must be less than 6cm (otherwise, the angle opposite the base would not be the single largest angle in the triangle). To reiterate, the two angles opposite the equivalent sides must be smaller than the angle opposite the base (otherwise the angle opposite 6cm side would not be the single largest angle).
c. The two equal sides must be longer than 3cm. Otherwise a closed triangle could not be formed (i.e., the lines would not connect).
d. Draw a diagram with the information we know:



AC = 6
3cm < AB < 6cm
3cm < BC < 6cm

e. Although we know the base, we know nothing about the height, BD. Without knowing that a definitive value for the height exists, we cannot calculate the area of the triangle.
f. Statement (1) is NOT SUFFICIENT.

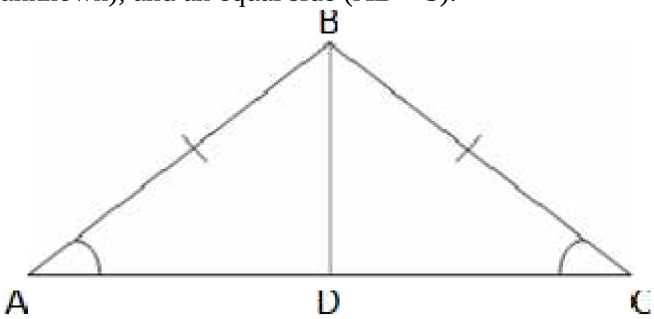
(iii) Evaluate Statement (2) alone.

a. Since triangle X is an isosceles triangle, the perimeter is formed by adding two equal sides and a third side. Set up an equation to reflect this:
$2L + N = 16$ where L is the length of the equivalent sides of the triangle and N is the length of the other side.

b. There are many different combinations of L and N that would give a different area. Assume that N is the base:
If N = 4 and L = 6, then the height of the triangle (via the Pythagorean theorem) would be the square root of 32, which is 5.65
If N = 5 and L = 5.5, then the height of the triangle (via the Pythagorean theorem) would be the square root of 24, which is 4.89.
c. Without being able to determine that a definitive value for the base and height exists, we cannot calculate the area of the triangle.
d. Statement (2) is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2) together.

a. Write equations that are derived from the information in both Statements:
(1) Base = 6cm
(2) 2L + B = 16
b. Combine the two equations:
2L + 6 = 16
L = 5
c. We now have a right triangle formed by half the base (AD = 3), the height (BD = unknown), and an equal side (AB = 5).



AC = 6
AD = 3
AB = BC = 5
Note: Since the triangle is isosceles, the height (or line BD) must perfectly bisect the base. This is because angles A and C are equal and sides AB and BC are equal. Further, angle D is right since it is the height and the height is by definition a right angle.
d. Through the Pythagorean theorem, BD must equal 4:
$(AD)^2 + (BD)^2 = (AB)^2$
$9 + (BD)^2 = 25$
$(BD)^2 = 16$
BD = 4
e. The area of triangle X must be (1/2)Base*Height = 3(4) = 12
f. Statements (1) and (2), when taken together, are SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, but Statements (1) and (2), when taken together are SUFFICIENT, answer C is correct.

20. Option A

(i) The mode is the number that appears the most times in the set, and the median is the middle number when the set is sorted from least to greatest. The range is the largest value in the set minus the smallest value.

(ii) Given that there is only one mode, at least two of the numbers must be equal. In particular, either all the numbers are equal, or two of the numbers are equal and the third is different.

(iii) Evaluate Statement (1) alone.

- (a) First, assume that all three of the numbers in the set are equal. Represent this set by {A, A, A}. The range is equal to A - A = 0, because the highest number and the lowest number in the set are the same.
- (b) By Statement (1), the range is equal to the median. Since the range is 0, the median is 0. Now the set can be represented by {A, 0, A}, since the median is the middle number. But all the numbers in the set are the same, so A = 0 and the set can be represented by {0, 0, 0}. The mode is 0 because this is the number that appears the most times.
- (c) Therefore, when all three numbers in the set are equal, the mode is equal to the range.
- (d) Now, assume that only two of the numbers in the set are equal. The set can be represented as {A, A, B}, where A is not the same as B. When sorted from least to greatest, the set become either {A, A, B} or {B, A, A}. The median of both of these sets is A. By Statement (1), the range is A because the range is equal to the median. The mode is also A, because it appears in the set more times than B.
- (e) Therefore, when only two numbers in the set are equal, the mode is equal to the range.
- (f) Whether three numbers in the set are equal, or only two, the mode is always equal to the range.
- (g) Statement (1) is SUFFICIENT.

(iv) Evaluate Statement (2) alone.

- (a) First, assume that all three of the numbers in the set are equal. Represent this set by {A, A, A}. Statement (2) says that A = 2A, because A is both the largest number and the smallest number in the set. The only way A = 2A is if A = 0.
- (b) When A = 0, the set becomes {0, 0, 0}. The range is 0 - 0 = 0, and the mode is 0. Thus, the mode equals the range when all three numbers are equal.
- (c) Now, assume that only two of the numbers in the set are equal. The set can be represented as {A, A, B}, where A is not the same as B. When sorted from least to greatest, the set become either {A, A, B} or {B, A, A}. Statement (2) says that A = 2B, or B = 2A, depending on whether A or B is larger.
- (d) Assuming A is the larger number, A = 2B. For example, A = 8 and B = 4. Then the sorted set becomes {4, 8, 8}. In this case, the range is 8 - 4 = 4. The mode is 8, because it appears the most times in the set. The mode is NOT equal to the range.
- (e) Now, assume B is the larger number, B = 2A. For example, A = 3 and B = 6. Then the sorted set becomes {3, 3, 6}. In this case, the range is 6 - 3 = 3. The mode is 3. The mode is equal to the range in this case.
- (f) Whether or not the mode is equal to the range depends on whether A or B is larger. Therefore the answer cannot be determined from Statement (2) alone.
- (g) Statement (2) is NOT SUFFICIENT.

(v) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

21. Option C

(i) Evaluate statement (1) alone

a) First solve the equation from Statement (1) for P.
$Q^2 - 2 = P$
$Q^2 = P + 2$
Q = Sqrt(P + 2)

b) Since P is a prime less than 10, try the possible values for P.

P	Sqrt(P + 2)
2	2
3	Sqrt(5)
5	Sqrt(7)
7	3

c) As seen in the table, when P = 2 or P = 7, then Q is prime. Otherwise, Q is not a prime number, nor even an integer. Whether Q is prime or not depends on P, so the question cannot be answered.

d) Statement (1) is NOT SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) Since Q + 2 is not prime, let L be a number that is not prime, and L = Q + 2. Examine two different examples for L.
b) Let L = 4. This implies that Q = 2, which is a prime number.
c) Let L = 8. This implies that Q = 6, which is not a prime number.
d) Whether Q is prime or not depends on L, so the question cannot be answered.
e) Statement (2) is NOT SUFFICIENT.

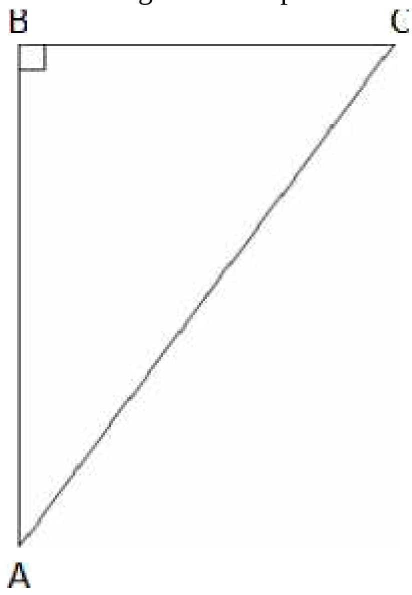
(iii) Evaluate Statement (1) and (2) together.

a) From the table above, only two values of P allow for Q to be an integer--which is demanded in Statement (2). In particular, when P = 2, Q = 2 and when P = 7, Q = 3.
b) So far, there are only two possibilities for values of Q. Now apply Statement (2). Q + 2 may not be a prime number. Thus, the case where Q = 3 is no longer a possibility because Q + 2 = 5, which is a prime number.
c) The only possibility that remains after applying Statements (1) and (2) is Q = 2. Thus, Q is a prime number.
d) Statement (1) and (2) together are SUFFICIENT.

(iv) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT yet Statements (1) and (2), when taken together, are SUFFICIENT, answer C is correct.

22. Option D

(i) Draw a diagram of the problem with the information from the question:



AC = 15

You are dealing with a right-triangle since a right angle will be formed by going straight north and then turning straight east.

(ii) Evaluate Statement (1) alone.

b) Translate the information from Statement (1) into algebra:
(AB)/(BC) = 4/3
3(AB) = 4(BC)

c) Set up a Pythagorean theorem equation:
$(AB)^2 + (BC)^2 = (AC)^2$
$(AB)^2 + (BC)^2 = (15)^2$

d) You now have two equations and two unknowns:
Equation (1): $(AB)^2 + (BC)^2 = (15)^2$
Equation (2): 3(AB) = 4(BC)

e) With two unique equations and two unknowns, a solution must exist. With this solution, you can subtract 15 from the detour distance and arrive at an answer.

f) Statement (1) is SUFFICIENT.

g) Note: You should not do these calculations on the test since they are not necessary for determining sufficiency. However, to demonstrate that there is a solution, we show how you would arrive at a numerical answer:
$(AB)^2 + (BC)^2 = (15)^2$
$(AB)^2 + (.75AB)^2 = (15)^2$
{rearrange Equation 2, solving for BC and substitute in BC=.75AB}
$(AB)^2(1 + .75^2) = 225$ {factor out $(AB)^2$}
$(AB)^2 = 144$
AB = 12
Substitute Back into Equation 2:
3(12) = 4(BC)
BC = 9

h) With these two distances, you can calculate the distance traveled on the detour.
Direct Route: 15
Detour: 9 + 12 = 21
Extra Distance: 21 - 15 = 6

(iv) Evaluate Statement (2) alone.

(a) Set up a Pythagorean theorem equation:
$(AB)^2 + (BC)^2 = (AC)^2$
$(AB)^2 + (BC)^2 = (15)^2$
(b) You are told that AB = 12. Substitute this information in and solve for BC.
$(12)^2 + (BC)^2 = (15)^2$
$(BC)^2 = 81$
BC = 9
(c) Statement (2) is SUFFICIENT.
(d) Note: You should not do these calculations on the test since they take up time and are not necessary for determining sufficiency. However, to demonstrate that there is a solution, we show how you would arrive at a numerical answer:
Direct Route: 15
Detour: 9 + 12 = 21
Extra Distance: 21 - 15 = 6

(v) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

23. Option B
(i) There are two possible cases (or conditions) under which the product of X and Y could be positive:
Case (1): Positive(Positive) = Positive
Case (2): Negative(Negative) = Positive

(ii) Evaluate Statement (1) alone.

b) Since Y is raised to an odd exponent, the sign of the base (i.e., the sign of Y) is the same as the sign of the entire expression (i.e., the sign of Y^5).
c) There is no way of distinguishing whether we are in Case (1) or (2) and the answer to the resulting question of whether X + Y is negative can be different depending on the chosen values of X and Y. Consider two examples, one from each case.
d) Case (1):
X = 100
Y = 2
XY = (100)(2) = 200 = Positive
$X > Y^5$
X + Y = 100 + 2 = 102 = Positive
e) Case (2):
X = -10
Y = -2
XY = (-10)(-2) = 20 = Positive
$X > Y^5$
X + Y = (-10) + (-2) = -12 = Negative
f) Since there is no way to determine whether X + Y is positive, Statement (1) is not sufficient.
g) Statement (1) is NOT SUFFICIENT.

(iv) Evaluate Statement (2) alone.

a. Y^6 must be a positive number since, even if Y were negative, raising it to an even exponent would make the entire quantity positive.
b. Substituting this into the information given in Statement (2):
$X > Y^6$

X > (positive number)
X must be positive since any number that is larger than a positive number is itself positive.

c. Since X is positive, in order for XY to be positive, Y must also be positive (i.e., we are dealing with Case (1) from above). Consequently, a positive number (i.e., X) plus a positive number (i.e., Y) must itself be positive.
X + Y = ?
Positive + Positive = Positive
We can definitively answer "no" to the original question.
d. Statement (2) is SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT but Statement (2) alone is SUFFICIENT, answer B is correct.

24. Option C

(i) A number divided by 5 will be an odd integer if and only if that number contains only odd factors, one of which is 5. In other words, there are two conditions under which x divided by 5 will be an odd integer:
(1) x is a multiple of 5
(2) x contains only odd factors

(ii) Evaluate Statement (1) alone.

b) If x contains only odd factors, there is no guarantee that one of those factors is 5. Consequently, there is no guarantee that x will be divisible by 5.
c) For example:
9 = 3*3 --> not an odd integer when divided by 5 since 5 is not a factor
21 = 3*7 --> not an odd integer when divided by 5 since 5 is not a factor
15 = 3*5 --> an odd integer when divided by 5 since 5 is a factor
105 = 3*7*5 --> an odd integer when divided by 5 since 5 is a factor
d) Statement (1) alone is NOT SUFFICIENT.

(iv) Evaluate Statement (2) alone.

a. Simply because x is a multiple of 5 does not guarantee that x only contains odd factors. Consequently, there is no guarantee that x is divisible by 5.
5 = 5*1 --> an odd integer when divided by 5 because 5 is a factor and there are only odd factors
15 = 5*3 --> an odd integer when divided by 5 because 5 is a factor and there are only odd factors
25 = 5*5 --> an odd integer when divided by 5 because 5 is a factor and there are only odd factors
20 = 5*4 --> not an odd integer when divided by 5 because there is at least one even factor
30 = 6*5 --> not an odd integer when divided by 5 because there is at least one even factor
b. Statement (2) alone is NOT SUFFICIENT.

(v) Evaluate Statements (1) and (2) together.

a. With only odd factors and x as a multiple of 5 (i.e., with 5 as a factor), you know that x divided by 5 must be an odd number since the two conditions laid out earlier are fulfilled.
b. Consider the following examples:
5 = 5*1 --> an odd integer when divided by 5

15 = 5*3 --> an odd integer when divided by 5
25 = 5*5 --> an odd integer when divided by 5
35 = 5*7 --> an odd integer when divided by 5

c. Statements (1) and (2), when taken together, are SUFFICIENT.

(vi) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, but Statements (1) and (2), when taken together, are SUFFICIENT, answer C is correct.

25. Option A

(i) Simplify the equation:
Is $(2^{y+z})(3^x)(5^y)(7^z) < (90^y)(14^z)$?
Simplified: is $(2^{y+z})(3^x)(5^y)(7^z) < ((2*5*3*3)^y)((7*2)^z)$?
Simplified: is $(2^{y+z})(3^x)(5^y)(7^z) < (2^y)(5^y)(3^{2y})(7^z)(2^z)$?
Simplified: is $(2^{y+z})(3^x)(5^y)(7^z) < (2^{y+z})(5^y)(3^{2y})(7^z)$?
Cancel out 2^{y+z}, 5^y, and 7^z
Simplified: is $3^x < 3^{2y}$?

(ii) Evaluate Statement (1) alone.

b) Statement (1) says that x = 1. So, plug that information in and work from there.
Simplified Question: is $3^1 < 3^{2y}$ where y is a positive integer?
Further Simplified: is 1 < 2y where y is a positive integer?
c) At this point, some students can see that Statement (1) is SUFFICIENT. However, a more thorough analysis is provided just to be clear.
d) Since x and y are given as positive integers, the smallest possible value for y is 1. In this case 1 < 2(1). Since the inequality held true when y=1, it will hold true for any legal value of y since y will only get larger and x will not change.
e) Thus, 3^x will always be less than 3^{2y}.
Statement (1) is SUFFICIENT.

(iv) Evaluate Statement (2) alone.

a. Statement (2) says y = 1 and x and z are positive integers. So, plug that information and work from there.
Is $3^x < 3^{2(1)}$?
Or: is $3^x < 3^2$?
Or: is x < 2?
b. Since the only restriction on x is that it is a positive integer, x could be 1 (in which case the inequality would be true and the answer to the question would be "Yes") or, x could be 2 (in which case the inequality would not be true and the answer to the question would be "No").
c. Since different answers to the question "is x < 2?" are possible, there is no definitive answer to the question. Statement (2) is NOT SUFFICIENT.

(v) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

26. Option A

(i) Simplify the original question by factoring:
$x^2 + 2x > x^2 + x$
2x > x
x > 0
Simplified question: is x > 0?

(ii) Evaluate Statement (1) alone.

b) When dealing with a number that is raised to an even exponent, it is important to remember that the sign of the base number can be either positive or negative (i.e., if $x^2 = 16$, x = -4 and 4). Moreover, it is important to remember that raising a fraction to a larger exponent makes the resulting number smaller:
$(1/2)^2 > (1/2)^3 > (1/2)^4$

c) There are three possible cases:
Case (1): $x < 0$
If x were negative, x^{odd} would be negative while x^{even} would be positive. This would make x^{odd} {=negative} < x^{even} {=positive}, which is an explicit contradiction of Statement (1). As a result, we know x cannot be negative. Statement (1) is SUFFICIENT. At this point, you should not keep evaluating since you know that Statement (1) provides enough information to answer the question "is x > 0?"

Case (2): $0 < x < 1$
In this case, based upon what was shown above, for $x^{\text{odd integer}} > x^{\text{even integer}}$ to hold true, *odd integer* must be less than *even integer*.

Case (3): $x > 1$
This case is the opposite of Case (2). In other words, for $x^{\text{odd integer}} > x^{\text{even integer}}$, the *odd integer* must be greater than the *even integer*.

d) Since Statement (1) eliminates the possibility of x being a negative number, we can definitively answer the question: is x > 0?

e) Statement (1) alone is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

f) Factor $x^2 + x - 12 = 0$
$(x - 3)(x + 4) = 0$
$x = 3, -4$

g) Since x can be either positive or negative, Statement (2) is not sufficient.

h) Statement (2) alone is NOT SUFFICIENT.

(iv) Since Statement (1) alone is SUFFICIENT but Statement (2) alone is NOT SUFFICIENT, answer A is correct.

27. Option E

(i) In evaluating this problem, it is important to keep in mind the list of possible prime numbers:
7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53

(ii) Evaluate Statement (1) alone.

a) The prime numbers between 15 and 34, not-inclusive, include:
17, 19, 23, 29, 31

b) Since there is no definitive information about the value of X, we do not know how many prime numbers exist between 7 and X.
If X = 17, there would be 2 prime numbers between 7 and X (i.e., 11 and 13).
If X = 18, there would be 3 prime numbers between 7 and X (i.e., 11, 13, and 17).
If X = 21, there would be 4 prime numbers between 7 and X (i.e., 11, 13, 17, and 19).
There is not enough information to definitively answer the question.

c) Statement (1) alone is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) List the multiples of 11 and their sums (stopping when the sum is no longer less than 7).
x = 11; sum of digits is 1+1 = 2
x = 22; sum of digits is 2+2 = 4
x = 33; sum of digits is 3+3 = 6
x = 44; sum of digits is 4+4 = 8, which is too high so x cannot be greater than 33.

b) Since X can be 11, 22, or 33, there are different possible answers to the question of how many prime numbers are there between the integers 7 and X:
If X = 11, there would be 0 prime numbers *between* 7 and X.
If X = 22, there would be 4 prime numbers *between* 7 and X (i.e., 11, 13, 17, and 19).
There is not enough information to definitively answer the question.

c) Statement (2) alone is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2) together.

a) Putting Statements (1) and (2) together, X must meet the following conditions:
(1) 15 < X < 34
(2) X = 11, 22, 33
This means that possible values for X include:
X = 22 or 33

b) The two possible values for X give different answers to the original question:
If X = 22, there would be 4 prime numbers between 7 and X (i.e., 11, 13, 17, and 19).
If X = 33, there would be 7 prime numbers between 7 and X (i.e., 11, 13, 17, 19, 23, 29, and 31).

c) Statements (1) and (2), even when taken together, are NOT SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT, Statement (2) alone is NOT SUFFICIENT, and Statements (1) and (2), even when taken together, are NOT SUFFICIENT, answer E is correct.

28. Option B

(i) If "the value of every item in Country X plummeted by 50% from 1990 to 1995," the value in 1995 would be 100%-50% = 50% of the value in 1990.
Translate the information in the question into an algebraic equation.
$P_{95} = (.5)P_{90}$; $P_{90} = ?$

(ii) Evaluate Statement (1) alone.

a) Statement (1) says that P_{93} = $30.

b) It may be tempting to assume that the value of the book changed the same amount each year. If this were true, Statement (1) would be sufficient since $30 would be the result of the value falling an equal percent for a known number of years. But, you cannot make this assumption. All you know is that the value in 1995 was half the value in 1990 and the value in 1993 was $30. It is possible that the value could have risen substantially from 1990 to 1993, only to fall dramatically enough during 1993, 1994, and 1995 that the value decreased by 50% from 1990 to 1995.

c) Consider the following two examples, which are both possible under the constraints of Statement (1) yet give different values for P_{90}.
(1) P_{90} = $15 and the value doubled to P_{93} = $30, only to fall to P_{95} = $7.5
(2) P_{90} = $25 and the value grew 20% to P_{93} = $30, only to fall to P_{95} = $7.5
Since both examples satisfy the conditions (i.e., P_{93} = $30 and $P_{95} = (.5)P_{90}$) yet produce different values for P_{90}, Statement (1) is not definitive.

d) Statement (1) alone is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Translate Statement (2) into algebra.
$(.5)P_{95} = P_{2000}$
$(.5)P_{95} = \$25$
Therefore: $P_{95} = \$50$
b) The original question states the following relationship:
$P_{95} = (.5)P_{90}$
Since we know that $P_{95} = \$50$, by substitution, we also know that:
$\$50 = (.5)P_{90}$
Therefore: $P_{90} = \$100$
Statement (2) is SUFFICIENT.

(iv) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

29. Option A
(i) Evaluate statement (1) alone
a) Substitute Z into the equation:
$10x + 10y + 16x^2 + 25y^2 = 10 + Z$
$10x + 10y + 16x^2 + 25y^2 = 10 + (4x)^2 + (5y)^2$
$10x + 10y + 16x^2 + 25y^2 = 10 + 16x^2 + 25y^2$
$10x + 10y = 10$
$10(x+y) = 10$
$x + y = 1$

b) Statement (1) is SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) Substitute the information you know (i.e, x = 1) into the equation:
$10x + 10y + 16x^2 + 25y^2 = 10 + Z$
$10(1) + 10y + 16(1)^2 + 25y^2 = 10 + Z$
$10y + 25y^2 + 10 + 16 = 10 + Z$
$25y^2 + 10y + 16 - Z = 0$
b) At this point, we reach a wall. Since we do not know what Z equals, we cannot solve for Y. Without a value for Y, we cannot find x + y.
c) Statement (2) is NOT SUFFICIENT.

(iii) Since Statement (1) alone is SUFFICIENT but Statement (2) alone is NOT SUFFICIENT, answer A is correct.

30. Option D
(i) Evaluate statement (1) alone

a) The equation in Statement (1) can be factored.
$x^2 + 4x + 4 = 0$
$(x + 2)(x + 2) = 0$
Consequently, x = -2.
b) With one specific value of x, the inequality can be definitively evaluated:
Is $-2|(-2)|^3 < (|-2|)^{-2}$?
c) Since this will give a definitive answer, the data are sufficient. (Note: Although the answer to the question here is yes, it does not need to be yes in order for sufficiency to exist. In other words, if the answer to our question were always no, that would be sufficient.) Statement (1) is SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) With the information in Statement (2), plug in the sign of x:
is (negative)(|negative|)^3 < (|negative|)^{negative}?
Simplified:
Is (negative)(positive)^3 < (positive)$^{(negative)}$?
Since a positive number raised to an odd exponent is always positive and (negative)(positive) = negative, we can simplify further:
Is (negative) < (positive)$^{(negative)}$?
Since a positive number raised to a negative number is simply a smaller positive number, we can simplify further:
Is (negative) < (positive)?

b) Statement (2) enables the question to be definitively answered. Statement (2) is SUFFICIENT.

(iii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

31. Option A

(i) A number is divisible by any of its prime factors or any combination of its prime factors. For a number to be divisible by four, it must have two 2s in its prime factorization since 2*2 = 4 and, if 4 is a factor of X, X will be divisible by 4.

(ii) Evaluate Statement (1) alone.

a) Since X has two 2s in its prime factorization, 4 must be a factor of X and, consequently, X must be divisible by 4. Statement (1) is SUFFICIENT.

b) If this seems too abstract, consider the following examples which show that whenever X has at least two 2s in its prime factorization (which it must as per Statement (1)), X is divisible by 4:
X = 4: has two 2s in its prime factorization and, as a result, is divisible by 4
X = 6: has only one 2 in its prime factorization and, as a result, is not divisible by 4
X = 8: has at least two 2s in its prime factorization and, as a result, is divisible by 4
X = 10: has only one 2 in its prime factorization and, as a result, is not divisible by 4

c) Since X cannot be 6, 10, etc. as these values do not have at least two 2s as prime factors (as is required by Statement (1)), X can only be 4, 8, etc. and will always be divisible by 4.

d) Statement (1) alone is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) If X is divisible by 2, you know that X must have at least one 2 in its prime factorization. However, you do not know that X has two 2s in its prime factorization and, as a result, you cannot be sure that X is divisible by 4.

b) If this seems too abstract, consider the following examples, all of which are divisible by 2 in keeping with the requirements of Statement (2):
X = 4: has two 2s in its prime factorization and, as a result, is divisible by 4
X = 6: has only one 2 in its prime factorization and, as a result, is not divisible by 4
X = 8: has at least two 2s in its prime factorization and, as a result, is divisible by 4
X = 10: has only one 2 in its prime factorization and, as a result, is not divisible by 4

c) Statement (2) alone is NOT SUFFICIENT.

(iv) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

32. Option B

(i) The phrase "how much more of substance X does she need than substance Y" can be translated into algebra as: X – Y

(ii) Evaluate Statement (1) alone.

a) Although X = 10, there is no information about Y. Consequently, we cannot determine the value of X - Y
b) Statement (1) is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Y = 4 and the ratio of X:Y = 5:2. Consequently, X = 10 and X - Y = 10 - 4 = 6
b) Statement (2) is SUFFICIENT.

(iv) Since Statement (1) alone is NOT SUFFICIENT but Statement (2) alone is SUFFICIENT, answer B is correct.

33. Option A

(i) Evaluate statement (1) alone

a) 40% of 200 is 80, so Michael sold 80 computers with Vista and less than 4GB of RAM.
b) These 80 computers represent 80% of the total computers Michael sold with Vista, so Michael sold a total of 100(=80/80%) computers with Vista.
c) $Vista_{total} = Vista_{<4GB} + Vista_{>4GB}$
$100 = 80 + Vista_{>4GB}$
$Vista_{>4GB} = 20$
d) Statement (1) alone is SUFFICIENT.
e) Note: At this point, you should not continue making calculations since you have determined sufficiency. However, to be complete, we included additional information you can deduce:
$Vista_{Total} + NoVista_{Total} = Total$
$100 + NoVista_{Total} = 200$
$NoVista_{Total} = 100$ and Michael sold a total of 100 computers without Vista.

	Vista	No Vista	Total
> 4GB RAM	20		
< 4GB RAM	80		
Total	100	100	200

(ii) Evaluate Statement (2) alone.

a) "50% of the 200 total computers that Michael sold had Vista" means that 100 computers had Vista. Consequently, 100 computers Michael sold did not have Vista.
b) "Of the computers that Michael sold without Vista, half had more than 4GB of RAM while the other half had less than 4GB of RAM" means that one-half of the 100 computers without Vista (i.e., 50) had more than 4GB of RAM while the other half (i.e., 50) had less than 4GB of RAM.
c) Put this information into a table:

	Vista	No Vista	Total

> 4GB RAM ?=		50	
< 4GB RAM		50	
Total	100	100	200

d) We cannot determine the number of computers sold both with Vista and more than 4GB of RAM, so Statement (2) is NOT SUFFICIENT.
e) Statement (2) alone is NOT SUFFICIENT.

(iii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

34. Option A

(i) Before evaluating Statements (1) and (2), it is extremely helpful to keep in mind that an odd number is the result of a sum of numbers with unlike parity. In other words: even + odd = odd. Since 17,283 is odd, the only way x + 17,283 will be odd is if x is even. Consequently, the simplified version of the question is: is x even?

(ii) Evaluate Statement (1) alone.

a) Statement (1) says that x - 192,489,358,935 is odd. Since there is only one way for a difference to be odd (i.e., if the parity of the numbers is different), Statement (1) implies that x is even (otherwise, if x were odd, x - 192,489,358,935 would be even). Since Statement (1) gives the parity of x, it is SUFFICIENT.
b) Statement (1) alone is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Statement (2) says that x/4 is not an even integer. It is important to note that this does not mean that x cannot be even (e.g., 6 is even yet 6/4 is not an even integer). Possible values of x include 2, 3, 6, 10, 11.

x	Parity of x	x/4
2	Even	Not an even integer
3	Odd	Not an even integer
6	Even	Not an even integer
10	Even	Not an even integer
11	Odd	Not an even integer

b) As this list indicates, there is no definitive information about the parity of x (e.g., 11 is odd and 10 is even). Consequently, Statement (2) is NOT SUFFICIENT.
c) Statement (2) alone is NOT SUFFICIENT.

(iv) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

35. Option B

(i) Evaluate statement (1) alone

a) The smallest possible value of n is 1 since n is a positive integer and the smallest positive integer is 1
b) You know that $z^2 > 1$, which is the smallest possible value of n. Possible values of z include any number whose absolute value is greater than 1. This does not provide enough information to answer the question definitively. Consider the following examples.
If z = -10 and n = 1, two values that are permissible since $(-10)^2 > 1$, then the answer to the original question is *yes* since 1 + 2 > -10.

However, if $z = 10$ and $n = 1$, two values that are permissible since $(10)^2 > 1$, then the answer to the original question is *no* since 1 + 2 is not greater than 10.
Statement (1) is NOT SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) Statement (2) can be re-arranged:
$z - n < 0$
$z < n$
Stated Differently: $n > z$
Since n is greater than z, n+2 will definitely be greater than z because n is a positive integer and it will only become larger.
In other words, let z = (a number less than n). You can be sure that n + 2 will definitely be greater than (a number less than n).
Statement (2) is SUFFICIENT.

(iii) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

36. Option C
(i) Since this is a distance-rate-time problem, begin with the core equation:
Distance = Rate(Time)
Note that there are two distance equations, one for traveling the expressway and the other for traveling the backroads.
$Distance_{express} = Rate_{express}(Time_{express})$
$Distance_{backroad} = Rate_{backroad}(Time_{backroad})$
(ii) In order to answer the question, you need to find the value of:
$Time_{express} - Time_{backroad}$

(iii) Evaluate Statement (1) alone.

a) Statement (1) says $Rate_{express} = Rate_{backroad} = 60$ mph.
b) Statement (1) also says that $2(Time_{backroad}) = 1$ hour
(Time is multiplied by 2 because the statement gives the time "to drive round-trip to and from work.")
$Time_{backroad} = 1/2$ hour.
c) Filling in all the information, you have the following:
$Distance_{express} = 60(Time_{express})$
$Distance_{backroad} = 60mph((1/2)\ hour) = 30$ miles
d) Without information concerning the distance or time to travel on the expressway, you cannot solve for $Time_{express}$. Consequently, Statement (1) is NOT SUFFICIENT.

(iv) Evaluate Statement (2) alone.

a) Statement (2) says that $2(Distance_{express}) = Rate_{express}((2/3)$ of an hour)
(Note that the distance is multiplied by two because Peter travels twice the distance when he goes "to and from work".)
So, $Time_{express} = 1/3$ of an hour.
b) Fill in the information that is known:
$Distance_{express} = Rate_{express}(1/3$ of an hour)
Without any information about $Time_{backroad}$, you cannot determine $Time_{express} - Time_{backroad}$. Statement (2) is NOT SUFFICIENT.

(v) Evaluate Statements (1) and (2) together.

a) Putting Statements (1) and (2) together, you know $Time_{backroad}$ from Statement (1) and you know $Time_{express}$ from Statement (2).
b) So, $Time_{express}$ - $Time_{backroad}$ = 1/3hour - 1/2hour or 20 minutes - 30 minutes = 10 minutes or 1/6 of an hour. Statements (1) and (2) together are SUFFICIENT.

(vi) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT yet Statements (1) and (2), when taken together, are SUFFICIENT, answer C is correct.

37. Option E

(i) The key insight to solving this problem is realizing that whether a and/or b are integers will have a significant influence on the logic of this problem. Further, it is important to not assume that a and b have to be integers. If a and b are not integers, the number of integers between them will be different than if a and b are integers. Consider two examples:
(1) Let a=20 and b=10
10 < {9 integers} < 20
(2) Let a=20.1 and b=10.1
10.1 < {11 integers} < 20.1

(ii) Evaluate Statement (1) alone.

a) If a and b are integers, 77 integers satisfy the inequality. For example, if a = 78 and b = 0, then the integers 1 through 77 satisfy the inequality.
0 < {77 integers} <78
b) However, a and b could both be non-integers, in which case there would be 78 integers that satisfy the inequality. For example, if a = 78.1 and b = 0.1, then the integers 1 through 78 satisfy the inequality.
0.1 < {78 integers} < 78.1
c) Since there is no way to determine how many integers satisfy the inequality, Statement (1) is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Statement (2) provides no definitive information about a and b. If a = 110 and b = 0, there are 109 integers that satisfy the inequality. However, if a = 111 and b = 0, there are 110 integers that satisfy the inequality. There is a limitless number of counter-examples since there are no closed-end boundaries to a and b. Consider the following counter-examples, each of which fits the stipulations of Statement (2) (i.e., a > 100 and b < 50):
0 < {199 integers} < 200
0.1 < {200 integers} < 200.1
0 < {249 integers} < 250
0 < {299 integers} < 300
b) Statement (2) is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2) together.

a) Taking Statements (1) and (2) together, there is still no resolution to the problem of whether a and b are integers. Consequently, the same problem that caused Statement (1) to be NOT SUFFICIENT will cause Statements (1) and (2), even when taken together, to be NOT SUFFICIENT.
b) To help see this, consider the following examples:
(1) Let a = 101 and b = 23, so a-b = 78 and a > 100, b < 50
23 < {77 integers} < 101

(2) Let a = 101.1 and b = 23.1, so a-b = 78 and a > 100, b < 50
23.1 < {78 integers} < 101.1

(v) Since Statements (1) and (2), even when taken together, are NOT SUFFICIENT, answer E is correct.

38. Option C

(i) Evaluate statement (1) alone

a) Cross-multiply:
   c/b = 2/d
   cd = 2b
b) Since b could be any integer, the value of cd cannot be definitively determined. For example, if b = 2, then cd = 4. However, if b = 3, then cd = 6.
c) Since we cannot determine the value of cd, Statement (1) is NOT SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) Simplify by dividing common terms:
   $b^3a^4c = 27a^4c$
   $b^3 = 27$; (divided by a^4c)
   b = 3
b) By knowing that b = 3, there is no information about the value of cd. (Do not make the mistake of importing the information from Statement (1) into your evaluation of Statement (2)).
c) Since we cannot determine the value of cd, Statement (2) is NOT SUFFICIENT.

(iii) Evaluate Statements (1) and (2) together.

a) Combining Statements (1) and (2), you know that b = 3 and cd = 2b.
b) By plugging b = 3 into cd=2b, you know that cd = 2(3) = 6. Combining Statements (1) and (2), you can find a definitive value of cd.

(iv) Statements (1) and (2), when taken together, are SUFFICIENT. Answer C is correct.

39. Option A

(i) Based upon the question, we can set up a few equations:
Equation (1): Sugar/Flour = 2/1

Since one cake could be made from 2 cups of sugar and 1 cup of flour (or different number of cups in the same ratio):
Equation (2): Sugar/(Total Ingredients) = 2/(2+1) = 2/3
Equation (3): Flour/(Total Ingredients) = 1/(2+1) = 1/3

(ii) Evaluate Statement (1) alone.

a) Since the cake requires 33 cups of ingredients, using Equation (2), we know that Total Ingredients = 33:
   Sugar/(Total Ingredients) = 2/3
   Sugar/33 = 2/3
   Therefore: Sugar = 22 cups.
b) Statement (1) is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Statement (2) does not provide any new information. Based upon the original question, we derived Equation (3). Statement (2) is merely a restatement of Equation (3).
b) Consider two examples:
If there were 10 cups of flour, the total amount of ingredients would be 30 cups and there would be 20 cups of sugar.
But, if there were 5 cups of flour, the total amount of ingredients would be 15 cups and there would be 10 cups of sugar.
c) Statement (2) is NOT SUFFICIENT since we cannot determine how many cups of sugar were used in the cake.

(iv) Since Statement (1) alone is SUFFICIENT but Statement (2) alone is NOT SUFFICIENT, answer A is correct.

40. Option C

(i) Note that this question asks for a specific number, not a ratio. Consequently, keep in mind that knowing y percent of the total staff is composed of women from outside the United States is not sufficient.

(ii) Evaluate Statement (1) alone.

a) If 25% of the staff are men, 75% must be women.
b)

	Men	Women	Total
From U.S.			
From Outside U.S.			
	.25(x)	.75(x)	x

c) There is not enough information to determine the number of women from outside the United States. Statement (1) alone is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Since 20 men from the U.S. represent 20% of the staff, the total staff is 100. We also know that there are 20 men from the U.S. and 2(20)=40 women from the U.S. for a total of 20+40=60 employees from the U.S. Consequently, 100-60=40 employees must be from outside the U.S.
b)

	Men	Women	Total
From U.S.	20	40	60
From Outside U.S.			40
			x=100

c) Since we cannot determine the breakdown of the 40 employees from outside the U.S., it is impossible to determine the number of women from outside the U.S.; Statement (2) alone is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2) together.

a) Fill in as much information as possible from Statements (1) and (2). We now know that there are a total .25(x)=.25(100)=25 men and .75(x)=.75(100)=75 women.
b)

	Men	Women	Total
From U.S.	20	40	60
From Outside U.S.	5	35	40

25   75   x=100

c) 35 members of the staff of Advanced Computer Technology Consulting are women from outside the United States.

(v) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, but Statements (1) and (2), when taken together, are SUFFICIENT, answer C is correct.

41. Option A

(i) Evaluate statement (1) alone

a) Take the cube root of both sides:
$8x^3 = 27y^3$
$2x = 3y$
b) Rearrange in order to find a ratio of 2x to y.
$2x/y = 3$
c) Consequently, 2x is 3 times y.
d) Statement (1) alone is SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) Take the square root of both sides:
$4x^2 = 9y^2$
$2x = 3y$
$2x/y = 3$
b) However, we must also consider that in taking the square root, a negative root is possible. To illustrate this, consider the following example:
Let x = 3 and y = 2 --> $4x^2 = 9y^2$
Let x = -3 and y = 2 --> $4x^2 = 9y^2$
Let x = -3 and y = -2 --> $4x^2 = 9y^2$
Let x = 3 and y = -2 --> $4x^2 = 9y^2$
c) In the four examples above, although $4x^2 = 9y^2$, there is no consistent ratio of 2x to y since the negative numbers cause ratios to be negative. Consequently, Statement (2) is NOT SUFFICIENT.

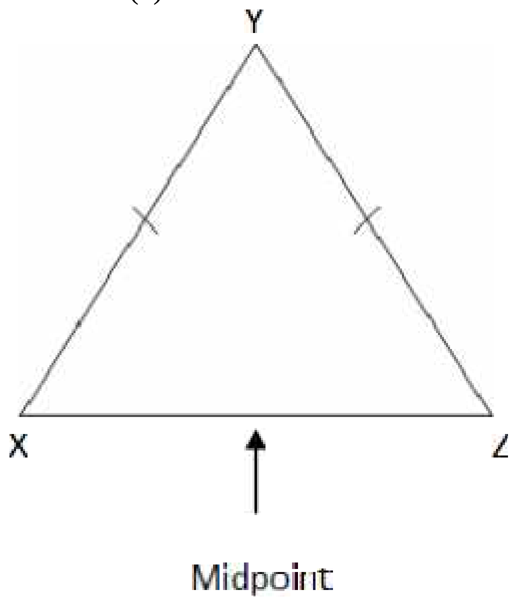
(iii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

42. Option B

(i) Evaluate statement (1) alone

a) It is possible that XZ is a straight line with Y as the midpoint, making ZY=YX.
b) However, just because ZY = YX does not mean Y must always be the midpoint; XYZ could be an equilateral triangle.

c) Statement (1) alone is NOT SUFFICIENT.



(ii) Evaluate Statement (2) alone.

a) By definition, the center of a circle is the midpoint of a diameter. Consequently, XZ runs through point Y and XY = YZ since both are radii and all radii must be the same length.
b) Statement (2) alone is SUFFICIENT.

(iii) Since Statement (1) alone is NOT SUFFICIENT but Statement (2) alone is SUFFICIENT, answer B is correct.

43. Option B
(i) Be aware that simply because you have two equations with two unknowns does not mean that a solution exists. You must have two unique equations with two unknowns in order for a solution to exist.

(ii) Evaluate Statement (1) alone.

a) There are two possible ways to solve this problem:
Method (1): Substitute b from Statement (1) into the original equation.
$15a + 6(5 – 2.5a) = 30$
$15a + 30 - 15a = 30$
$30 = 30$
$0 = 0$
Based upon this answer, the equation in Statement (1) is the equation in the original question solved for b. Consequently, we only have one equation and two unknowns. There is not enough information to determine a-b.

Method (2): Rearrange the equation in Statement (1) and subtract this equation from the original equation.
$b = 5 – 2.5a$
$b + 2.5a = 5$
$2.5a + b = 5$
Multiply by 6 so *b*'s cancel:15a + 6b = 30
This method also shows that the equation in Statement (1) is nothing more than the

original equation rearranged. Consequently, we only have one equation and two unknowns. There is not enough information to determine a-b.

b) Statement (1) is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Try to line up the two equations so that you can subtract them:
$9b = 9a – 81$
$81 + 9b = 9a$
$81 = 9a - 9b$
Statement (2) Equation: $9a - 9b = 81$
Original Question Equation: $15a + 6b = 30$
At this point, you can stop since you know that you have two unique equations and two unknowns. Consequently, there will be a solution for *a* and for *b*, which means there will be one unique value for a-b. Statement (2) is SUFFICIENT.

b) If you want to solve to see this (Note: Do not solve this in a test as it takes too much time and is not necessary):
Multiply (2) by 4: $36a - 36b = 324$
Multiply Original by 6: $90a + 36b = 180$

6*Original + 2*Statement(2): $(90a + 36a) + (36b + -36b) = 180 + 324$
$126a = 204$
$a = 4$

Solve for b:
$9b = 9(4) - 81 = -45$
$b = -5$

$a - b = 4 - (-5) = 4 + 5 = 9$

(iv) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

44. Option D

(i) Evaluate statement (1) alone

a) Try to solve for n:
$n^2 - 6n = -9$
$n^2 - 6n + 9 = 0$
$(n - 3)^2 = 0$
$n - 3 = 0$
$n = 3$
With one value for n, we can find a single value for $(n + 1)^2$

b) Statement (1) alone is SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) Expand the terms and simplify them:
$n^2 - 2n + 1 = n^2 – 5$
$-2n + 1 = -5$
$-2n + 6 = 0$
$6 = 2n$
$n = 3$
With one value for n, we can find a single value for $(n + 1)^2$

b) Statement (2) alone is SUFFICIENT.

(iii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

45. Option E

(i) Revenue = $Cost_{store\text{-}front}$*$Quantity_{store\text{-}front}$ + $Cost_{bulk}$*$Quantity_{bulk}$

(ii) Evaluate Statement (1) alone.

a) Store-Front: 75% of 100 packages is 75 packages at the store-front rate.
b) Bulk: The remainder (or 25%) of the 100 packages (i.e., 100-75 = 25) is 25 packages at the bulk-rate.
c) Without any dollar amounts (such as the cost of the bulk-rate and the cost of the store-front rate), it is impossible to calculate John's total revenue.
d) Statement (1) alone is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Translate Statement (2) into algebra:
Store-Front > 2(Bulk)
$4 > 2(Bulk)
Bulk < $2
b) Although Statement (2) tells us the dollar amount of each shipping rate, without information about the number of packages shipped at each rate, it is impossible to calculate John's revenue.
c) Statement (2) alone is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2) together.

a) Store-Front: 75 packages {from Statement (1)} shipped at $4 each {from Statement (2)} -> $300 in revenue from the store-front rate.
b) Bulk: 25 packages shipped at less than $2 each; no more than $50 in revenue from the bulk-rate.
c) However, you still cannot calculate the total revenue definitively.
Revenue = $Cost_{store\text{-}front}$*$Quantity_{store\text{-}front}$ + $Cost_{bulk}$*$Quantity_{bulk}$
Filling in what we found thus far:
Revenue = $4*75 + $Cost_{bulk}$*25
d) Statements (1) and (2), even when taken together, are NOT SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT, Statement (2) alone is NOT SUFFICIENT, and Statements (1) and (2), even when taken together are NOT SUFFICIENT, answer E is correct.

46. Option E

(i) Any positive integer that is divided by 2 will have a remainder of 1 if it is odd. However, it will not have a remainder if it is even.
N/2 --> Remainder = 0 if N is even
N/2 --> Remainder = 1 if N is odd

(ii) Evaluate Statement (1) alone.

a) If a number contains only odd factors, it will be odd (and will have a remainder of 1 when divided by 2). If a number contains at least one even factor, it will be even (and divisible by 2).

15 = 3*5 {only odd factors; not divisible by 2; remainder of 1}
21 = 3*7 {only odd factors; not divisible by 2; remainder of 1}
63 = 3*3*7 {only odd factors; not divisible by 2; remainder of 1}

30 = 3*5*2 {contains an even factor; divisible by 2}
42 = 3*7*2 {contains an even factor; divisible by 2}
50 = 5*5*2 {contains an even factor; divisible by 2}

b) Simply because "N contains odd numbers as factors" does not mean that all of N's factors are odd. Consequently, it is entirely possible that N contains an even factor, in which case N is even and N is divisible by 2. Possible values for N:
18 = 2*3*3 {contains odd factors, but is divisible by 2; remainder = 0}
30 = 2*5*3 {contains odd factors, but is divisible by 2; remainder = 0}
But:
27 = 3*3*3 {contains odd factors, but is not divisible by 2; remainder = 1}
15 = 3*5 {contains odd factors, but is not divisible by 2; remainder = 1}

c) Since some values of N that meet the conditions of Statement (1) are divisible by 2 while other values that also meet the conditions of Statement (1) are not divisible by 2, Statement (1) does not provide sufficient information to definitively determine whether N is divisible by 2.

d) Statement (1) alone is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Since "N is a multiple of 15", possible values for N include:
15, 30, 45, 60, 75, 90

b) Possible values for N give different remainders when divided by 2:
15/2 --> Remainder = 1
30/2 --> Remainder = 0
45/2 --> Remainder = 1
60/2 --> Remainder = 0
75/2 --> Remainder = 1
90/2 --> Remainder = 0

c) Since different legitimate values of N give different remainders when divided by 2, Statement (2) is not sufficient for determining the remainder when N is divided by 2.

d) Statement (2) alone is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2).

a) Since "N is a multiple of 15" and "N contains odd numbers as factors", possible values for N include:
15, 30, 45, 60, 75, 90

b) Adding Statement (1) to Statement (2) does not provide any additional information since any number that is a multiple of 15 must also have odd numbers as factors.

c) Possible values for N give different remainders when divided by 2:
15/2 --> Remainder = 1
30/2 --> Remainder = 0
45/2 --> Remainder = 1
60/2 --> Remainder = 0
75/2 --> Remainder = 1
90/2 --> Remainder = 0

d) Since different legitimate values of N give different remainders when divided by 2, Statements (1) and (2) are not sufficient for determining the remainder when N is divided by 2.

e) Statements (1) and (2), even when taken together, are NOT SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer E is correct.

47. Option D

(i) Do not be distracted by "X and Y are both positive integers whose combined factors include 3 and 7." The factors given do not allow you to conclude that X or Y is either odd or even. To conclude that X and Y are even, X and Y need to have at least one even factor. To conclude that X and Y are odd, X and Y must only have odd factors.

(ii) For X + Y + 1 to be odd, the sum X + Y must be even since adding one to an even integer makes it odd. Said algebraically:
X + Y + 1 = odd
X + Y = even

(iii) The sum of two integers will be even if and only if the parity of the two numbers is the same. In other words, odd + odd = even and even + even = even. However, the sum of two numbers of different parity is odd (i.e., odd + even = odd). Consequently, in order for X + Y = even, both X and Y must be of the same parity. There are two possibilities:
$X_{odd} + Y_{odd}$ = even
$X_{even} + Y_{even}$ = even

(iv) Evaluate Statement (1) alone.

a) A number is divisible by 2 if and only if it is even. Consider the following examples:
4 is even and divisible by 2
5 is not even and not divisible by 2
6 is even and divisible by 2
7 is not even and not divisible by 2
8 is even and divisible by 2
9 is not even and not divisible by 2
10 is even and divisible by 2
11 is not even and not divisible by 2

b) Since Statement (1) tells us that both X and Y are divisible by 2, both X and Y are even. Since X and Y have the same parity, the sum X + Y is even and the sum X + Y + 1 is odd; Statement (1) is SUFFICIENT.

c) Statement (1) alone is SUFFICIENT.

(v) Evaluate Statement (2) alone.

a) If you take a number and add 2, you do not change the parity of that number. Consider the following examples:
4 {i.e., even} + 2 = 6 {i.e., even}
5 {i.e., odd} + 2 = 7 {i.e., odd}
6 {i.e., even} + 2 = 8 {i.e., even}
7 {i.e., odd} + 2 = 9 {i.e., odd}
8 {i.e., even} + 2 = 10 {i.e., even}
9 {i.e., odd} + 2 = 11 {i.e., odd}

b) Statement (2) indicates that the parity of X and Y are the same since adding 2 to X will not change the parity of X.
X + 2 = Y
$Parity_X + 2 = Parity_Y$
$Parity_X = Parity_Y$
Statement (2) is SUFFICIENT.

c) Statement (2) alone is SUFFICIENT.

(vi) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

48. Option D

(i) Write out the formula for the mean and arrange it in several different ways so that you can spot algebraic substitutions:
Mean = (w + x + y + z + 10)/5
5*Mean = w + x + y + z + 10

(ii) Evaluate Statement (1) alone.

a) Translate each piece of information into algebra:
"the average (arithmetic mean) of w and y is 7.5"
(w + y)/2 = 7.5
w + y = 15

"the average (arithmetic mean) of x and z is 2.5"
(x + z)/2 = 2.5
x + z = 5

b) Combine the two equations by adding them together:
(x + z) + (w + y) = (5) + (15)
x + z + w + y = 15 + 5
w + x + y + z = 20

c) Substitute into the equation from the top:
Equation from top: 5*Mean = w + x + y + z + 10
5*Mean = 20 + 10 = 30
Mean = 6

d) Statement (1) alone is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Simplify the algebra:
-[-z - y -x - w] = 20
z + y + x + w = 20

b) This can be substituted into the mean formula:
z + y + x + w = 20
w + x + y + z = 20 {rearrange left side to make substitution easier to see}

Equation from top: 5*Mean = w + x + y + z + 10
5*Mean = (w + x + y + z) + 10
5*Mean = 20 + 10 = 30 {substitute information from Statement (2)}
Mean = 6

c) Statement (2) alone is SUFFICIENT.

(iv) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

49. Option A

(i) Simplify the question:
Since multiplying a number by 13 does not change its sign, the question can be simplified to: "is N a positive number?"

(ii) Evaluate Statement (1) alone.

a) Write out algebraically:
-21N = negative

21N = positive {divided by -1}
N = positive

b) Since N is a positive number, 13N will always be a positive number.
c) Statement (1) alone is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Any time you are dealing with a number raised to an even exponent, you must remember that the even exponent hides the sign of the base (e.g., $x^2 = 16$; x = 4 AND -4).
b) Solve the inequality:
$N^2 < 1$
$-1 < N < 1$ {take the square root, remembering that there is a positive and negative root}
c) Since N can be both positive (e.g., .5) or negative (e.g., -.5), Statement (2) is not sufficient.
d) Statement (2) alone is NOT SUFFICIENT.

(iv) Since Statement (1) alone is SUFFICIENT but Statement (2) alone is NOT SUFFICIENT, answer A is correct.

50. Option B

(i) Since the sum of the measure of the interior angles of a triangle equals 180 degrees, you can write the following equation:
The measure of angles A + B + C = 180

(ii) Evaluate Statement (1) alone.

a) Translate Statement (1) into algebra:
A + C = 120
b) Use the foundational triangle equation (i.e., all angles add up to 180):
A + B + C = 180
(A + C) + B = 180
Substitute A + C = 120 into the equation.
120 + B = 180
B = 60
c) It is impossible to determine the value of angle C. Angle A could be 60 degrees and angle C could be 60 degrees. However, angle A could be 20 degrees and angle C could be 100 degrees.
d) Statement (1) is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Translate Statement (2) into algebra:
A + B = 80
b) Use the foundational triangle equation (i.e., all angles add up to 180):
A + B + C = 180
(A + B) + C = 180
Substitute A + B = 80 into the equation.
80 + C = 180
C = 100
c) Statement (2) is SUFFICIENT.

(iv) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.

51. Option C

(i) The basic equation necessary for solving this problem is:
Distance = Rate*Time
D = RT
If Officer Johnson can prove that R, the driver's average rate or speed, exceeded 50 miles-per-hour, Officer Johnson can prove that the driver broke the speed limit. We must be able to find R in order to definitively answer the question of whether Officer Johnson's assertion is correct.

(ii) Evaluate Statement (1) alone.

a) Statement (1) indicates that D = 30 miles. Without information about T or R, we cannot find the value of R and, as a result, we cannot prove or disprove Officer Johnson's claim.
b) Statement (1) alone is NOT SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Statement (2) indicates that T = 30 minutes. Without information about D or R, we cannot find the value of R and, as a result, we cannot prove or disprove Officer Johnson's claim.
b) Statement (2) alone is NOT SUFFICIENT.

(iv) Evaluate Statements (1) and (2) together.

a) From Statement 1: D = 30 miles
b) From Statement 2: T = 30 minutes
c) Putting the information together, we can construct the following algebraic equation:
   D = RT
   30mil = R(30min)
   R = 1mil/1min = one mile per minute
   R = One mile per minute * 60 minutes per hour = 60 miles per hour.
d) Since we have a value for R, we can definitively judge the veracity of Office Johnson's claim.
e) Statements (1) and (2), when taken together, are SUFFICIENT.

(v) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, but Statements (1) and (2) when taken together are SUFFICIENT, answer C is correct.

52. Option D

(i) The value of a fraction is less than one if its numerator is smaller than its denominator. For example, 4/6 is less than one because 4 < 6. So, the question at hand can be simplified to: is z < n?

(ii) Evaluate Statement (1) alone.

a) Statement (1) can be re-arranged:
   z - n > 0
   z > n
b) Since z > n, you can definitively answer *no* to the question: "is z < n?"
c) Statement (1) is SUFFICIENT.

(iii) Evaluate Statement (2) alone.

a) Based upon the question, since z-15 is a positive number, the following inequality must hold:
$z - 15 > 0$
$z > 15$
b) Statement (2) says:
$n < 15$
c) Since $z > 15$ and $n < 15$, you know that $z > n$
d) You can definitively answer *no* to the question: "is $z < n$?"
e) Statement (2) is SUFFICIENT.

(iv) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is SUFFICIENT, answer D is correct.

53. Option A
(i) Evaluate statement (1) alone
a) Simplify the inequality:
$4x^2 – 8x > (2x)^2 – 7x$
$4x^2 – 8x > 4x^2 – 7x$
$-8x > -7x$
$-8x + 8x > -7x + 8x$
$0 > x$
$x < 0$
b) Since X is less than zero, X is a negative number. This means that negative X is a positive number since multiplying a negative number by a negative number (i.e., -1) results in a positive number.
c) Statement (1) alone is SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) Simplify the inequality:
$x + 2 > 0$
$x > - 2$
b) Since we cannot be sure whether X is negative (e.g., -1) or positive (e.g., 2), we cannot be sure whether negative X is positive or negative.
c) Statement (2) alone is NOT SUFFICIENT.

(iii) Since Statement (1) alone is SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, answer A is correct.

54. Option C
(i) Evaluate statement (1) alone
a) Statement (1) simply says that B > 10. It provides no information about the value of A, making a comparison between B and A impossible.
b) If B = 12 and A = 5, then the answer to the question "is B > A?" would be yes. However, if B = 15 and A = 20, then the answer to the question "is B > A?" would be no.
c) Since different legitimate values of A and B produce different answers to the question, Statement (1) is NOT SUFFICIENT.
d) Note: Some students are thrown off by setting A = 20 or A = 5. You can do this in evaluating whether Statement (1) alone is sufficient since there is nothing in Statement (1) that prevents this. However, A cannot be 20 in evaluating statement 2 because Statement (2) clearly says that A must be less than 10. But, for now we are evaluating Statement (1).

(ii) Evaluate Statement (2) alone.

a) Statement (2) simply says that A < 10. It provides no information about the value of B, making a comparison between B and A impossible.
b) If B = 12 and A = 5, then the answer to the question "is B > A?" would be yes. However, if B = 1 and A = 9, then the answer to the question "is B > A?" would be no.
c) Since different legitimate values of A and B produce different answers to the question, Statement (2) is NOT SUFFICIENT.
d) Note: Some students are thrown off by setting B = 1 or B = 12. You can do this in evaluating whether Statement (2) alone is sufficient since there is nothing in Statement (2) that prevents this.

(iii) Evaluate Statements (1)and (2) together.

a) When taking Statements (1) and (2) together, you know:
B > 10 and A < 10
b) So, you know that B > A. Statements (1) and (2), when taken together, are SUFFICIENT.

(iv) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT yet Statements (1) and (2), when taken together, are SUFFICIENT, answer C is correct.

55. Option C
(i) Factor the original equation:
xn – ny – nz = n(x - y - z)
(ii) If we know the value of both *n* and *x* - *y* - *z*, we can determine the value of xn – ny – nz.

(iii) Evaluate Statement (1) alone.

a) Since x – y – z = 10, based upon the above factoring:
xn – ny – nz = n(10)
However, we do not know the value of *n* so we cannot solve for the value of xn – ny – nz.
b) Statement (1) is NOT SUFFICIENT.

(iv) Evaluate Statement (2) alone.

a) Since n = 5, based upon the above factoring:
xn – ny – nz = 5(x - y - z)
However, we do not know the value of *x* - *y* - *z* so we cannot solve for the value of xn – ny – nz.
b) Statement (2) is NOT SUFFICIENT.

(v) Evaluate Statements (1) and (2) together.

a) Since n = 5 and x - y - z = 10, based upon the above factoring:
xn – ny – nz = n(x - y - z)=5(10)=50
b) Statements (1) and (2), when taken together, are SUFFICIENT.

(vi) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is NOT SUFFICIENT, but Statements (1) and (2), when taken together, are SUFFICIENT, answer C is correct.

56. Option B

(i) Evaluate statement (1) alone

a) Make a list of even numbers and evaluate whether they are prime:
   2: Prime
   4: Not Prime
   6: Not Prime
   8: Not Prime
   10: Not Prime
b) Every single even number except 2 is not a prime number. However, since Statement (1) enables X to be prime (e.g., 2) and not prime (e.g., 4, 6, 8, 10, …), Statement (1) is NOT SUFFICIENT.
c) Statement (1) alone is NOT SUFFICIENT.

(ii) Evaluate Statement (2) alone.

a) List the possible values of X, remembering that X is a positive integer such that $1 < X < 4$:
   X = 2: Prime Number
   X = 3: Prime Number
   Since all possible values of X given the parameters in Statement (2) are prime, Statement (2) is SUFFICIENT.
b) Statement (2) alone is SUFFICIENT.

(iii) Since Statement (1) alone is NOT SUFFICIENT and Statement (2) alone is SUFFICIENT, answer B is correct.